THE MAXIMAL D=5 SUPERGRAVITIES

BdW, H. Samtleben, M. Trigiante
(in preparation)

Ungauged D=5 supergravity (Cremmer)

- toroidal reduction from IIB or 11D supergravity
- symmetry $E_{6(6)}^{\text{rigid}} \times USp(8)^{\text{local}}$

  ↑ R symmetry, maximal compact subgroup of $E_{6(6)}$
  composite connection

- scalar fields parametrize $E_{6(6)}/USp(8)$

$$\mathcal{V}(x) \longrightarrow \underset{\in E_{6(6)}}{g}\ \mathcal{V}(x)\ \underset{\in USp(8)}{h^{-1}(x)}$$

when fixing $USp(8)$ gauge

$\mathcal{V}(x)$: coset representative

instead: keep local $USp(8)$

- field content

	$e_\mu{}^a$	$\psi_\mu{}^i$	$A_\mu{}^M$	χ^{ijk}	$\mathcal{V}$
$USp(8)$	1	8	1	48	$\overline{27}$
$E_{6(6)}$	1	1	$\overline{27}$	1	27

$\left.\begin{matrix}\overline{27}\\27\end{matrix}\right\}42$

known gaugings :

- $SO(6)$, $SO(p,6-p)$ Günaydin, Romans, Warner
- $CSO(p,q,r)$ $p+q+r=6$ Fre et al , dWST
- Scherk-Schwarz dWST, Hull et al , Ferrara et al

For maximal supergravities gaugings can be analyzed by group-theoretical methods (dWST)

This enables one to investigate the viability of a gauging prior to constructing the Lagrangian. The aim is to write a 'universal' Lagrangian which encompasses all gaugings. The gaugings are then encoded in an EMBEDDING TENSOR which defines the gauge group embedding in the duality group ($E_{6(6)}$). The embedding tensor must satisfy two group-theoretical constraints to ensure that a corresponding gauge invariant and supersymmetric Lagrangian exists.

However, ungauged Lagrangian is not unique ! vector-tensor duality

When retaining all 27 vector fields, gauging may lead to charged vector fields that do <u>not</u> belong to the adjoint representation of the gauge group

Inconsistent !

Example

$$E_{6(6)} \to SO(6) \times SL(2)$$

$$\overline{27} \to (15,1) + (2,\bar{6})$$

SO(6) gauge fields

charged vector fields must be dualized to tensors

$E_{6(6)}$ covariance lost

NB: similar to 4-dim where Lagrangian has a small invariance group than $E_{7(7)}$

e.g. $SL(8)$, $E_{6(6)} \times SO(1,1) \ltimes T^{27}$, $SU^*(8)$

Novel solution:

preserve $E_{6(6)}$ covariance by

choosing vector fields in $\overline{27}$ and tensor fields in 27 — at the same time

+ additional gauge invariance to balance degrees of freedom

EMBEDDING TENSOR $\Theta_M{}^\alpha$

rank Θ = dim gauge group

generators $X_M = \Theta_M{}^\alpha \, t_\alpha$

X_M: 27 of $E_{6(6)}$

t_α: adjoint 78 of $E_{6(6)}$

gauge fields $A_\mu{}^M X_M$ → covariant derivatives & field strengths

$$[t_\alpha, t_\beta] = f_{\alpha\beta}{}^\gamma \, t_\gamma \qquad E_{6(6)} \text{ closure}$$

closure gauge group:

$$[X_M, X_N] = f_{MN}{}^P \, X_P$$

$$\Rightarrow \quad \boxed{\Theta_M{}^\alpha \Theta_N{}^\beta f_{\alpha\beta}{}^\gamma = f_{MN}{}^P \, \Theta_P{}^\gamma}$$

not explicit expression for $f_{MN}{}^P$

Jacobi identity

$$f_{[MN}{}^Q f_{P]Q}{}^R \, \Theta_R{}^\alpha = 0 \qquad \text{only projected !}$$

in the $\overline{27}$ representation (gauge fields)

$$(X_M)_N{}^P \Theta_P = \Theta_M{}^\beta t_{\beta N}{}^P \, \Theta_P{}^\alpha$$

$$= - f_{MN}{}^P \, \Theta_P{}^\alpha$$

i.e. $$X_M \sim \left(\begin{array}{c|c} -f_M & * \\ \hline \emptyset & * \end{array} \right)$$

do not vanish

not subject to Jacobi id. !

Quadratic constraint (closure)

$$C_{MN}{}^{\alpha} \equiv f_{\beta\gamma}{}^{\alpha}\, \Theta_M{}^{\beta} \Theta_N{}^{\gamma} + (t_{\beta})_N{}^{P}\, \Theta_M{}^{\beta} \Theta_P{}^{\alpha} = 0$$

$E_{6(6)}$ covariant

or

$$[X_M, X_N] + X_{MN}{}^{P} X_P = 0$$

in $\overline{27}$:
$$X_{MP}{}^{R} X_{NR}{}^{Q} - X_{NP}{}^{R} X_{MR}{}^{Q} + X_{MN}{}^{R} X_{RP}{}^{Q} = 0$$

$\Leftrightarrow$ $\Theta_M{}^{\alpha}$ gauge invariant

T-tensor

$$\mathcal{V}(x) \in E_{6(6)} \quad \rightarrow \quad E_{6(6)}/USp(8)$$

scalars: $\mathcal{V}_M{}^{\underline{N}}(x)$ $\qquad$ M rigid $E_{6(6)}$

$\underline{N}$ local $USp(8) \subset E_{6(6)}$

$$T_{\underline{M}\,\underline{N}}{}^{\underline{P}}(\Theta,\phi) = \mathcal{V}^{-1}{}_{\underline{M}}{}^{M}\, \mathcal{V}^{-1}{}_{\underline{N}}{}^{N}\, \mathcal{V}_P{}^{\underline{P}}\, X_{MN}{}^{P}$$

$\Theta \times \Theta$ constraint is $E_{6(6)}$ covariant

$\rightarrow$ induces $T \times T$ constraint

T is tensor in $H_R = USp(8)$ field dependent

Θ is tensor in $G = E_{6(6)}$ constant

Θ treated as a 'spurion'

REPRESENTATION CONSTRAINT (supersymmetry)

$$D_\mu \to g\, A_\mu{}^M \underbrace{\theta_M{}^\alpha t_\alpha}_{X_M}$$

induces in $\delta_{susy} \mathcal{L}$ through $[D_\mu D_\nu]$

$$g\,(\bar\psi_\mu \epsilon)\; F_{\mu\nu}{}^M\, \mathcal{V}_M{}^{\underline{M}}\; T_{\underline{M}} \;\Big|_{USp(8)}$$

$$g\,(\bar\chi \epsilon)\; F_{\mu\nu}{}^M\, \mathcal{V}_M{}^{\underline{M}}\; T_{\underline{M}} \;\Big|_{E_{6(6)}/USp(8)}$$

$\uparrow$ T tensor

cancelled by new terms

$$\mathcal{L}_{extra} \sim g \left\{ A_1\, \bar\psi \psi + A_2\, \bar\psi \chi + A_3\, \bar\chi \chi \right\}$$

A_1, A_2, A_3 $USp(8)$ tensors (masslike terms)

must cancel against T (after $\delta_g \psi_\mu$, $\delta_g \chi$ variation)

matching representations wrt $E_{6(6)}$

$$T \in 351 \xrightarrow[USp(8)]{} \underset{A_1}{\underset{\uparrow}{36}} + \underset{A_2 \sim A_3}{\underset{\uparrow}{315}}$$

$$\theta_M{}^\alpha \sim T : 27 \times 78 = 27 + 351 + \overline{1728}$$

should vanish.

d	G	H	T
7	SL(5)	USp(4)	$10 \times 24 = 10 + 15 + 40 + 175$
6	SO(5,5)	USp(4) × USp(4)	$16 \times 45 = 16 + 144 + 560$
5	$E_{6(6)}$	USp(8)	$27 \times 78 = 27 + 351 + 1728$
4	$E_{7(7)}$	SU(8)	$56 \times 133 = 56 + 912 + 6480$
3	$E_{8(8)}$	SO(16)	$248 \times 248 = 1 + 248 + 3875 + 27000 + 30380$

$\Theta_M{}^\alpha$

$$\mathcal{R}_{fund} \times \mathcal{R}_{adj} \to \mathcal{R}_{fund} + \mathcal{R}_\Theta + \mathcal{R}'$$

known

$$(t_\alpha)_M{}^N \, \Theta_N{}^\alpha = 0 \qquad (t_\beta t^\alpha)_M{}^N \, \Theta_N{}^\beta = k \, \Theta_M{}^\alpha$$

Conjecture :

$\Theta_M{}^\alpha$ { linear representation constraint
quadratic closure constraint

define all the permissible gaugings.

! NO other conditions

quadratic constraint takes care of the order - g^2 variations of the action.

$$\Theta \times \Theta \to (351 \times 351)_{symm}$$

$$= \underbrace{\overline{27} + 1728}_{} + 351' + 7722 + 17550 + 34398$$

vanish owing to quadratic constraint

use d_{MNP}, d^{MNP} $E_{6(6)}$-invariant symmetric tensors

$$d_{MPQ}\, d^{NPQ} = \delta_M{}^N \quad \text{(relative normalization)}$$

derive

$$X_{PQ}{}^M\, d^{NPQ} \equiv Z^{MN} \quad \in 351$$

$$Z^{MN} = -Z^{NM}$$

e.g. results

$$X_{(MN)}{}^P = d_{MNQ}\, Z^{PQ}$$

$$\left.\begin{array}{l} Z^{MN} X_N = 0 \\ X_{MN}{}^{[P} Z^{Q]N} = 0 \end{array}\right\} \text{equivalent to } \Theta \times \Theta \text{ constraint !}$$

etc

follows from fact that X_M belongs to a single irreducible representation.

Special $E_{6(6)}$ basis

decompose

$$V^M = (V^A, V^a, V^u) \qquad V^a, V^u \perp \Theta_M{}^\alpha \sim \Theta_A{}^\alpha$$

$$V_M = (V_A, V_a, V_u) \qquad V_A, V_u \perp Z^{MN} \sim Z^{uv}$$

$A = 1, 2, .., r$

$a = r+1, r+2, \dots, 27-p$

$u = 28-p, \dots, 27$ (p even)

$$Z^{MN}\Theta_N{}^\alpha = 0$$

$$X_{AN}{}^P = \begin{pmatrix} -f_{AB}{}^C & h_{AB}{}^c & C_{AB}{}^w \\ 0 & 0 & C_{Ab}{}^w \\ 0 & 0 & D_{Av}{}^w \end{pmatrix}$$

row indices $N = B, b, v$

column indices $P = C, c, w$

[AB] antisymmetric

+ Jacobi identity

no Jacobi identity

$$\mathcal{F}_{\mu\nu}{}^A = \partial_\mu A_\nu{}^A - \partial_\nu A_\mu{}^A - g f_{BC}{}^A A_\mu{}^B A_\nu{}^C$$

$$\mathcal{F}_{\mu\nu}{}^a = \partial_\mu A_\nu{}^a - \partial_\nu A_\mu{}^a + g h_{BC}{}^a A_\mu{}^B A_\nu{}^C$$

gauge algebra

$$[\tilde{X}_A, \tilde{X}_B] = f_{AB}{}^C \tilde{X}_C - h_{AB}{}^a \tilde{X}_a$$

(dim is odd)

central extension

acts only on gauge fields

Vector - Tensor Duality

Last column in $X_{MN}{}^{P}$?

- convert $A_\mu{}^{u} \rightarrow$ tensors (Townsend, Pilch, Van Nieuwenhuizen Günaydin, Romans, Warner)

  but: affects $E_{6(6)}$ structure and
  requires a new Lagrangian for (almost) every gauging

- instead, exploit $Z^{MN} X_{NP}{}^{Q} = 0$, $X_{(MN)}{}^{P} = d_{MNQ} Z^{PQ}$, etc

introduce novel vector-tensor gauge invariance

$$A_\mu{}^{M} \in \overline{27} \qquad B_{\mu\nu M} \in 27$$

$$\delta A_\mu{}^{M} = \partial_\mu \Lambda^M - g\, X_{[PQ]}{}^{M} \Lambda^P A_\mu{}^{Q} - g\, Z^{MN} \Xi_{\mu N}$$

does not satisfy Jacobi id.
up to $\propto Z$ terms

tensor gauge tr.

D_μ is <u>invariant</u> under tensor transformation

$$[D_\mu , D_\nu] = -g\, \mathcal{F}_{\mu\nu}{}^{M} X_M \qquad \text{'defines' } \mathcal{F}_{\mu\nu}{}^{M}$$

$$\delta \mathcal{F}_{\mu\nu}{}^{M} = -g\, X_{NP}{}^{M} \Lambda^N \mathcal{F}_{\mu\nu}{}^{P}$$
$$+ g\, Z^{MN} \{ -2\, \partial_{[\mu} \Xi_{\nu]N} + \underbrace{\cdots\cdots}_{\text{complicated}} \}$$

Define

$$\mathcal{H}_{\mu\nu}{}^{M} \equiv \mathcal{F}_{\mu\nu}{}^{M} + g\, Z^{MN} B_{\mu\nu N}$$

transforms covariantly

$$\delta \mathcal{H}_{\mu\nu}{}^{M} = -g\, X_{PN}{}^{M} \Lambda^P \mathcal{H}_{\mu\nu}{}^{N}$$

$\Rightarrow$ fixes $\delta B_{\mu\nu M} = 2\partial_{[\mu}\Xi_{\nu]M} + \cdots\cdots$

only $Z^{MN} B_{\mu\nu N}$ will appear in $\mathcal{L}$

$\rightarrow$ effectively p tensor fields

after tensor gauge-fixing $27-p$ vector fields

universal Chern-Simons / B kinetic term

$$\mathcal{L} \propto \varepsilon^{\mu\nu\rho\sigma\tau} \times$$

$$g Z^{MN} B_{\mu\nu M}\Big[D_\rho B_{\sigma\tau N} + 4 d_{NPQ} A_\rho{}^P\big(\partial_\sigma A_\tau{}^Q + \tfrac{1}{3} g X_{[RS]}{}^Q A_\sigma{}^R A_\tau{}^S\big)\Big]$$

$$+ \tfrac{8}{3} d_{MNP}\Big[A_\mu{}^M \partial_\nu A_\rho{}^N \partial_\sigma A_\tau{}^P - \tfrac{3}{4} g X_{[QR]}{}^M A_\mu^N A_\nu{}^Q A_\rho{}^R\big(\partial_\sigma A_\tau{}^P + \tfrac{1}{5} g X_{[ST]}{}^P A_\sigma{}^S A_\tau{}^T\big)\Big]$$

<u>Note</u>. many more structures than e.g. in $SO(6)$ case

. supersymmetry has not been used directly

Supersymmetry variations

$$\delta e_\mu{}^a = \bar\epsilon_i \gamma^a \psi_\mu{}^i$$

$$\delta \mathcal{V}_M{}^{ij} = i\, \mathcal{V}_M{}^{kl} \left[4\, \Omega_{p[k} \bar\chi_{lmn]} \epsilon^p + 3\, \Omega_{[kl} \bar\chi_{mn]p} \epsilon^p \right] \Omega^{mi} \Omega^{nj}$$

$$\delta A_\mu{}^M = \left[2i\, \Omega^{ik} \bar\epsilon_k \psi_\mu{}^j + \bar\epsilon_k \gamma_\mu \chi^{ijk} \right] \mathcal{V}_{ij}{}^M$$

$$\delta B_{\mu\nu M} = \mathcal{V}_M{}^{ij} \left[4\, \bar\psi_{[\mu i} \gamma_{\nu]} \epsilon^k \Omega_{jk} - i\, \bar\chi_{ijk} \gamma_{\mu\nu} \epsilon^k \right]$$
$$+ 2 d_{MNP}\, A_{[\mu}{}^N\, \delta(\epsilon) A_{\nu]}{}^P$$

$$\delta \psi_\mu{}^i = \left(\partial_\mu \delta^i{}_j + Q_{\mu j}{}^i - \tfrac14 \omega_\mu{}^{ab} \gamma_{ab} \delta^i{}_j \right) \epsilon^j$$
$$+ i \left[\tfrac16 \left(\gamma_{\mu\nu\rho} \mathcal{H}^{\nu\rho\, ij} - 4 \gamma^\nu \mathcal{H}_{\mu\nu}{}^{ij} \right) + g\, \gamma_\mu A_1{}^{ij} \right] \Omega_{jk} \epsilon^k$$

$$\delta \chi^{ijk} = - i\, \gamma^\mu P_\mu{}^{ijkl} \Omega_{lm} \epsilon^m$$
$$- \tfrac34 \gamma^{\mu\nu} \left[\mathcal{H}_{\mu\nu}{}^{[ij} \epsilon^{k]} - \tfrac13 \Omega^{[ij} \mathcal{H}_{\mu\nu}{}^{k]m} \Omega_{mn} \epsilon^n \right]$$
$$+ g\, A_2^{l,ijk} \Omega_{lm} \epsilon^m$$

$\mathcal{V}_M{}^{ij}$, $\mathcal{V}_{ij}{}^M$ coset representative $E_{6(6)}/USp(8)$

$$\mathcal{H}_{\mu\nu}{}^{ij} = \mathcal{H}_{\mu\nu}{}^M \mathcal{V}_M{}^{ij}$$

$\boxed{A_1 \oplus A_2 \quad : \quad T \text{ tensor}}$ $\quad 351 \to 36 + 315$

uniform closure

$$\left[\delta(\epsilon_1), \delta(\epsilon_2) \right] = \xi^\mu D_\mu + \delta_{USp(8)} + \delta_{\text{vector}} + \delta_{\text{tensor}}$$

consistent with vector/tensor system !

CONCLUSIONS

- universal Lagrangian with the gauging encoded in the embedding tensor $\Theta \sim Z$
  $\rightarrow$ masslike terms + potential expressed in T tensor (dressed Θ)

- Θ: 2 constraints
  - representation (supersymmetry) $\overline{351}$
  - closure (group property) $\Theta \times \Theta$ no $\overline{27} + 1728$

  universal + ensure $g + g^2$ variations

- follows general pattern (established for $d=3,4$)
  no additional conditions

- without explicitly constructing the Lagrangian one can determine the viability of a certain gauging

- compare with $SO(6)$, $SO(p,6-p)$, $CSO(p,q,r)$ gaugings

$$E_{6(6)} \rightarrow (C)SO \times SL(2)$$

$$\overline{27} \rightarrow (15,1) + (6,2) \qquad \text{gauge fields}$$

$$M \begin{cases} [IJ] \rightarrow A, a \\ I\alpha \rightarrow u \end{cases}$$

$$Z^{uv} \propto \Theta^{IJ}\, \varepsilon^{\alpha\beta}$$

$$\Theta_{IJ,K}{}^{L} \propto \delta_{[I}^{L}\, \Theta_{J]K}$$

$$\Theta_{IJ} = \Theta_{JI} \rightarrow (\underbrace{+\cdots+}_{p}\ \underbrace{-\cdots-}_{q}\ \underbrace{0\cdots0}_{r})$$