GAUGING EXTENDED SUPERGRAVITIES

Zlatibor, August 2004

III Summerschool in modern mathematical physics.

① introduction

② gauging supergravity

③ maximal supergravity

④ the maximal $D=5$ supergravities

(work in preparation, dWST)

gauging: supergravity deformations that are relevant for, e.g.

- M/string theory, supergravity vacua
- moduli stabilization
- adS/CFT correspondence
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GAUGING EXTENDED SUPERGRAVITIES

① Introduction

– What is meant by gauging supergravity?

Deformations of supergravity theories by switching on a gauge coupling g associated with a certain gauge group.

– Relevance

M-theory compactifications

Kaluza-Klein compactifications → gauge fields

+ modifications & extensions

flux compactifications

Scherk-Schwarz reductions

etc.

Consider D-dimensional spacetime compactified to

$$\mathcal{M}_D \longrightarrow \mathcal{M}_d^{\text{spacetime}} \times \mathcal{M}_{D-d}^{\text{internal}}$$

$$\begin{array}{cc} \mathcal{M}_d & \mathcal{M}_{D-d} \\ \downarrow & \downarrow \\ x^\mu & y^m \end{array}$$

vielbein + vector/tensor gauge fields

D-dimensional spacetime diffeomorphisms

$$\xi^M(x,y) \begin{cases} \xi^\mu(x) & \text{spacetime diffs} \\ \xi^m(x,y) & \text{internal (gauge) transformations} \end{cases}$$

with further appropriate restrictions

D-bein

$$E_M{}^A(x,y) = \begin{pmatrix} e_\mu{}^\alpha & B_\mu{}^m e_m{}^a \\ 0 & e_m{}^a \end{pmatrix}$$

gauge choice

diffs: $\delta E_M{}^A = \partial_M \xi^N E_N{}^A + \xi^N \partial_N E_M{}^A$

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$
$$+ \; g_{mn}\,(dy^m + B_\mu{}^m dx^\mu)(dy^n + B_\nu{}^n dx^\nu)$$

$$g_{\mu\nu} = e_\mu{}^\alpha e_\nu{}^\beta \eta_{\alpha\beta}$$

$$g_{mn} = e_m{}^a e_n{}^b \eta_{ab}$$

decompositions ensure covariant tensors in d dimensions tensor calculus

$$\delta e_\mu{}^\alpha = \partial_\mu \xi^\nu e_\nu{}^\alpha + \xi^\nu \partial_\nu e_\mu{}^\alpha$$
$$+ \xi^m \partial_m e_\mu{}^\alpha$$

$$\delta B_\mu{}^m = \partial_\mu \xi^\nu B_\nu{}^m + \xi^\nu \partial_\nu B_\mu{}^m$$
$$+ \mathcal{D}_\mu \xi^m$$
$$\hookrightarrow \partial_\mu \xi^m - B_\mu{}^n \partial_n \xi^m + \xi^n \partial_n B_\mu{}^m$$

$$\delta e_m{}^a = \xi^\mu \partial_\mu e_m{}^a$$
$$+ \xi^n \partial_n e_m{}^a + \partial_m \xi^n e_n{}^a$$

→ d-dimensional vielbein + vector + scalar

expand fields over internal manifold

$$\Phi(x,y) = \sum_A \phi^A(x) \; Y^A(y)$$

$B_\mu{}^m$ gauge fields associated with the isometries of $\mathcal{M}^{\text{internal}}_{D-d}$

i.e.

$$\delta \mathring{g}_{mn}(y) = \partial_m \mathring{\xi}^p \mathring{g}_{np} + \partial_n \mathring{\xi}^p \mathring{g}_{mp} + \mathring{\xi}^p \partial_p \mathring{g}_{mn}$$
$$= \mathring{D}_m \mathring{\xi}_n + \mathring{D}_n \mathring{\xi}_m$$
$$= 0$$

$$\Rightarrow \mathring{\xi}^m(y) \propto K_I^m(y) \qquad \text{Killing vectors}$$

isometry algebra:

structure constants

$$K_I^{\,n}\,\partial_n K_J^{\,m} - K_J^{\,n}\,\partial_n K_I^{\,m} = f_{IJ}{}^K\, K_K^{\,m}$$

$$\mathcal{B}_\mu{}^m(x,y) \sim \mathcal{B}_\mu{}^I(x)\; K_I^{\,m}(y)$$

$$\xi^m(x,y) \sim \xi^I(x)\; K_I^{\,m}(y)$$

$$\delta \mathcal{B}_\mu{}^I = \mathcal{D}_\mu \xi^I = \partial_\mu \xi^I - \mathcal{B}_\mu{}^J f_{JK}{}^I\, \xi^K$$

nonabelian gauge fields

gauge group → isometry group

extensions with vector/tensor gauge fields

e.g. abelian vector field $\hat{A}_M(x,y)$

decompose

$$A_\mu = \hat{A}_\mu - \mathcal{B}_\mu{}^m \hat{A}_m$$

$$A_m = \hat{A}_m$$

again: $\hat{A} = A_m\,(dy^m + \mathcal{B}_\mu^m\, dx^\mu) + A_\mu\, dx^\mu$

$$\delta A_\mu = \partial_\mu \xi^\nu A_\nu + \xi^\nu \partial_\nu A_\mu + \xi^m \partial_m A_\mu + \mathcal{D}_\mu \Lambda$$

$\hookrightarrow (\partial_\mu - \mathcal{B}_\mu{}^n \partial_n)\Lambda$

$$\delta A_m = \xi^\mu \partial_\mu A_m + \partial_m \xi^n A_n + \xi^n \partial_n A_m + \partial_m \Lambda$$

Fluxes:

$$A_m = -\tfrac{1}{2} f_{mn}\, y^n \quad \to \quad \partial_m A_n - \partial_n A_m = f_{[mn]}$$

flux quantized : $f_{[mn]}$ constant

+ fluctuations (toroidal case)

$$A_m(x,y) = -\tfrac{1}{2} f_{mn}\, y^n + \tilde{A}_m(x)$$

$$\delta \tilde{A}_m(x) = \xi^\mu \partial_\mu \tilde{A}_m - \tfrac{1}{2} f_{[mn]}\, \xi^n(x) + \lambda_m$$

λ_m constant

toroidal $\xi^m(x,y) = \xi^m(x)$

$$\Lambda(x,y) = \lambda(x) + \lambda_m y^m$$

flux induces charge

entangling with graviphoton $B_\mu{}^m$

Scherk-Schwarz $\quad \partial_m \to M_m$

$$M_m = e^{-M_n y^n}\, \partial_m\, \underbrace{e^{M_n y^n}}_{\substack{\text{abelian subgroup} \\ \subset \text{ duality group} \\ \text{in } D \text{ dimensions}}}$$

$\Longrightarrow$ Low energy effective supergravity theories with "certain" gauge groups

② Gauging supergravity (generic)

in d dimensions
irrespective of origin

dW Samtleben Trigiante
hep-th/0212239
0311224, 0311225
04mmddd to appear

CONFLICT

gauge invariance $\Longleftrightarrow$ supersymmetry

generically:

$$\mathcal{L} = \cdots + \bar{\psi}_\mu \Gamma^{\mu\nu\rho} D_\nu \psi_\rho \Rightarrow \bar{\psi}_\mu [D_\nu D_\rho] \epsilon$$

$$\cdots + \bar{\chi} \not{D} \chi + \cdots \Rightarrow \bar{\chi} [\not{D}, \not{D}] \epsilon$$

$$[D_\mu, D_\nu] = \text{'usual terms'} + g F_{\mu\nu}$$

$$\delta\mathcal{L} \sim g F_{\nu\rho} \bar{\psi}_\mu \epsilon + g F_{\mu\nu} \bar{\chi} \gamma^\mu \epsilon$$

new variations! violate supersymmetry

order g

add $\mathcal{L}_g$ masslike terms

$$g A_1 \bar{\psi}_\mu \gamma^{\mu\nu} \psi_\nu + g A_2 \bar{\psi}_\mu \gamma^\mu \chi + g A_3 \bar{\chi} \chi$$

$$A_1 + A_2 + A_3 = T\text{-tensor}$$

requires
$$\left.\begin{array}{l} \delta\psi_\mu \propto g A_1 \gamma_\mu \epsilon \\ \delta\chi \propto g A_2 \epsilon \end{array}\right\}$$ new susy variations

$T = A_1 \oplus A_2 \oplus A_3$ tensor (field dependent) wrt H_R

$\boxed{\text{order - } g^2}$

$$\delta \mathcal{L} \sim g^2 (A_1^2 + A_2^2)\, \bar{\epsilon}\, \psi_\mu + g^2 (A_1 + A_3) A_2\, \bar{\epsilon}\, \chi$$

cancelled (?) by potential

$$\mathcal{L} \sim - g^2 V$$

$$g^2 V \sim -g^2 \left(|A_1|^2 - |A_2|^2 \right)$$

from 'matter' variation

from gravitino variation

unique deformation ?

for low susy extension + superpotential.

V: <u>not</u> positive definite

full susy: $\left.\begin{array}{l} A_2 = 0 \\ A_1 A_1^\dagger \propto \mathbb{1} \end{array}\right\}$ at (adS) stationary point.

<u>CENTRAL QUESTION</u> :

What are the conditions on the gauge group in order to preserve supersymmetry ?

Furthermore

methodology may depend sensitively
on spacetime dimension and # supersymmetries

does there exist a 'universal' Lagrangian
(of Yang Mills) with gauging encoded in

eg Killing vectors, moment maps, T tensor
covariant derivatives + Field strengths?

typical problem:

ungauged Lagrangian is ambiguous
because of

vector / tensor duality

electric / magnetic duality

different Lagrangians may have
different symmetry groups

symmetries of field equations are,
however, the same!

but gauge groups are to be embedded
in the symmetry group of the
Lagrangian!

③ Maximal supergravities

- obtained by toroidal compactifications of 11 D or IIB supergravity
- one dimensionful coupling constant
- unique supermultiplets of massless states
- field representations ambiguous because of vector/tensor dualities
- scalar fields parametrize homogeneous space nonlinear sigma model with target space

G/H R-symmetry group

nonlinearly realized

described in terms of $\mathcal{V}(x) \in G$

map spacetime $\to G$

$\mathcal{V}(x) \to g\, \mathcal{V}(x)\, h^{-1}(x)$

rigid G

local $H = H_R$

composite connections

physical scalar fields = $\dim[G] - \dim[H]$

gauge fixed $\mathcal{V}(x)$: coset representative

G-symmetry nonlinearly realized

[practical: use $\mathcal{V}$ without gauge fixing

linearly realized symmetries]

some examples

D	H	G	dim[H]		# scalars		dim[G]	# vectors
3	$SO(16)$	$E_{8(8)}$	120	+	128	=	248	*
4	$(S)U(8)$	$E_{7(7)}$	63	+	70	=	133	28
5	$USp(8)$	$E_{6(6)}$	36	+	42	=	78	27
6	$USp(4)\times USp(4)$	$SO(5,5)$	20	+	25	=	45	16
7	$USp(4)$	$SL(5)$	10	+	14	=	24	10
8	$U(2)$	$SL(3)\times SL(2)$	4	+	7	=	11	7

gauge group $\subset G$

dim[gauge group] $<$ dim[G]

$\leq$ # vector fields

how to embed the gauge group into G ?

historically use KK evidence

D=11	$\longrightarrow$ S^7	: D=4 max sg	SO(8) gauge group	'82
D=11	$\longrightarrow$ S^4	: D=7 max sg	SO(5) gauge group	'84
IIB	$\longrightarrow$ S^5	: D=5 max sg	SO(6) gauge group	'86

(Hull : noncompact versions or contractions)

new developments since 2000.

for maximal supergravity: gaugings defines the only supersymmetric deformations

(exception: IIA massive sg)