Rolling tachyon &

open-closed string duality

A rolling tachyon is a classical solution of SFT which represents the evolution in time of the tachyon field $T(x^0)$. (Sen)

In classical field theory

$$T(x^0) = \lambda \cosh x^0$$

$T(0) = \lambda$
$T'(0) = 0$

In CFT

$$-\frac{1}{2\pi}\int_{\Sigma} d^2z\, \partial_z X^0 \partial_{\bar{z}} X^0 + \tilde{\lambda}\int_{\partial\Sigma} dt \cosh X^0(t)$$

with $\tilde{\lambda} \simeq \lambda$.

Wick rotate $X_0 \to iX$ and study boundary state $|B\rangle$ perturbed by

$$\tilde{\lambda}\int dt \cos X(t)$$

After inverse-Wick rotating, the relevant part of $|B\rangle$ is

$$|B\rangle_{c=1} \sim \left[f(X^0(0)) + \alpha_{-1}^0 \tilde{\alpha}_{-1}^0 \, g(X^0(0)) \right] |0\rangle$$

where

$$f(x^0) = \frac{1}{1+e^{x^0}\sin\tilde{\lambda}\pi} + \frac{1}{1+e^{-x^0}\sin\tilde{\lambda}\pi} - 1$$

and

$$g(x^0) = \cos(2\pi\tilde{\lambda}) + 1 - f(x^0)$$

Interpretation

$$T_{00} = K\left(f(x^0) + g(x^0)\right) = K\left(\cos(2\pi\tilde{\lambda}) + 1\right)$$

$$T_{0i} = 0$$

$$T_{ij} = -2K\, f(x^0)\, \delta_{ij} \qquad K = \tfrac{1}{2}\tau_p$$

Comments.

- For $\tilde{\lambda} = \frac{1}{2}$ total energy vanishes, $f(x^0) = 0$

  (equivalent to array of D-branes at $x^0 = i(2n+1)\pi$)

- $0 < \tilde{\lambda} < \frac{1}{2}$ system evolves.

$$f(x^0) \xrightarrow[x^0 \to \infty]{} 0 \qquad g(x^0) \longrightarrow 1 + \cos(2\pi\tilde{\lambda})$$

So

$$T_{00} = \text{const} \qquad T_{ij} \longrightarrow 0$$

Tachyon matter

Effective Field Theory of Tachyonic Matter (Sen)

Proposal

$$S = -\int d^{p+1}x \, V(T) \sqrt{-\det A} \,, \qquad V(T) = e^{-\frac{T}{2}}$$

$$A_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu T \, \partial_\nu T \qquad \mu, \nu = 0, \dots, p$$

For spatially homogeneous time-dependent field configurations

$$T_{00} = e^{-\frac{T}{2}} \left(1 - (\partial_0 T)^2\right)^{-1/2}$$

Since T_{00} is conserved, $\partial_0 T \to 1$ as $T \to \infty$.

Solution for large x^0:

$$T = x^0 + C e^{-x^0} + O(e^{-2x^0})$$

Pressure:

$$p = e^{-\frac{T}{2}} \sqrt{1 - (\partial_0 T)^2} \simeq -\sqrt{2C}\, e^{-x^0}$$

No plane wave solution. Candidate to represent tachyon condensation.

Rolling tachyon-like solution in VSFT

Dead ends: take a lump solution

$$|\psi'\rangle = |\Xi\rangle_{25} \otimes |\Lambda'\rangle$$

$$|\Lambda'\rangle = \mathcal{N}' e^{-\frac{1}{2}\sum_{N,M\geq 0} a_N^+ S'_{NM} a_M^+} |\Omega_b\rangle$$

where: $a_N|\Omega_b\rangle = 0$ $\qquad \mathcal{N}' = \sqrt{3}\,\frac{V_{00}+b/2}{(2\pi b^3)^{1/4}}\sqrt{\det(1-X')\det(1+T')}$

To get time profile contract with $|x_0\rangle$

$$\langle x_0|\Lambda'\rangle = \sqrt{\frac{2}{b\pi}}\,\frac{\mathcal{N}'}{1+s'}\, e^{\frac{1}{b}\frac{s'-1}{s'+1}x_0^2 - \frac{2i}{\sqrt{b}}\frac{x_0 f_0}{1+s'} - \frac{1}{2}a^+W'a^+}$$

$$f_0 = \sum_{n=1}^{\infty} S'_{0n} a_n^+ \,, \qquad a^+W'a^+ = \sum_{n,m=1}^{\infty} a_n^+ W'_{nm} a_m^+$$

$$W'_{nm} = S'_{nm} - \frac{S'_{0n}S'_{0m}}{1+s'} \qquad s' = S'_{00}$$

Now do inverse-Wick rotation

$$x_0 \to i x_0 \qquad a_n^+ \to i a_n^+$$

$$\langle x_0|\Lambda'\rangle = \sqrt{\frac{2}{b\pi}}\,\frac{\mathcal{N}'}{1+s'}\, e^{\frac{1}{b}\frac{1-s'}{1+s'}x_0^2 + \frac{2i}{\sqrt{b}}\frac{x_0 f_0}{1+s'} + \frac{1}{2}a^+W'a^+}$$

BUT... $\quad -1 \leq s' \leq 1$

STAR ALGEBRA SPECTROSCOPY

PROBLEM: Diagonalize X, X^{12}, X^{21}, T

Use $K_1 = L_1 + L_{-1} \longrightarrow \mathcal{K}_1 = -(1+z^2)\frac{d}{dz}$

with properties

$$[K_1, X] = [K_1, X^{12}] = [K_1, X^{21}] = [K_1, T] = 0$$

Result:

$$K_1 v^{(\kappa)} = \kappa v^{(\kappa)}$$

$-\infty < \kappa < +\infty$

$v^{(\kappa)} = (v_1^{(\kappa)}, v_2^{(\kappa)}, \ldots)$

with

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} v_n^{(\kappa)} z^n = \frac{1}{\kappa}\left(1 - e^{-\kappa \tan^{-1} z}\right)$$

Then

$$X v^{(\kappa)} = \mu(\kappa) v^{(\kappa)}, \qquad \mu(\kappa) = -\frac{1}{1 + 2\cosh\frac{\pi\kappa}{2}}$$

$$X^{12} v^{(\kappa)} = \mu^{12}(\kappa) v^{(\kappa)}, \qquad \mu^{12}(\kappa) = -\left(1 + e^{\frac{\pi\kappa}{2}}\right)\mu(\kappa)$$

$$X^{21} v^{(\kappa)} = \mu^{21}(\kappa) v^{(\kappa)}, \qquad \mu^{21}(\kappa) = -\left(1 + e^{-\frac{\pi\kappa}{2}}\right)\mu(\kappa)$$

$$T v^{(\kappa)} = \tau(\kappa) v^{(\kappa)}, \qquad \tau(\kappa) = -e^{-\frac{\pi|\kappa|}{2}}$$

Remark: $-\frac{1}{3} \leq \mu(\kappa) < 0$, spectrum doubly degenerate except for $\mu(0) = -\frac{1}{3}$

Now define combinations

$$\hat{x}_\kappa = \frac{i}{\sqrt{2}}\left(e_\kappa - e_\kappa^+\right) = \sqrt{2}\sum_{n=1}^{\infty} v_{2n}(\kappa)\sqrt{2n}\,\hat{x}_{2n}$$

$$\hat{y}_\kappa = \frac{i}{\sqrt{2}}\left(o_\kappa - o_\kappa^+\right) = -\sqrt{2}\sum_{n=1}^{\infty}\frac{v_{2n-1}(\kappa)}{\sqrt{2n-1}}\,\hat{p}_{2n-1}$$

there are also

$$\hat{z}_\kappa = \frac{1}{\sqrt{2}}\left(e_\kappa + e_\kappa^+\right)$$

$$\hat{w}_\kappa = \frac{1}{\sqrt{2}}\left(o_\kappa + o_\kappa^+\right)$$

The eigenvalues x_κ, y_κ

$$\hat{x}_\kappa |x_\kappa\rangle = x_\kappa |x_\kappa\rangle \quad , \quad \hat{y}_\kappa |y_\kappa\rangle = y_\kappa |y_\kappa\rangle$$

are the Moyal conjugate coordinates

$$\boxed{[x_\kappa, y_{\kappa'}]_* = i\,\theta_\kappa\,\delta(\kappa-\kappa')} \qquad \theta_\kappa = 2\,\mathrm{th}\frac{\pi\kappa}{4}$$

Moyal product for string fields:

$$|\Psi\rangle \longrightarrow \Psi(\{x_{2n}\},\{x_{2n+1}\}) \longrightarrow \tilde{\Psi}(\{x_{2n}\},\{p_{2n-1}\}) \longrightarrow \Psi^M(x_\kappa, y_\kappa)$$

$$\Psi(\{x_{2n}\},\{x_{2n+1}\}) = \langle X(\sigma)|\Psi\rangle$$

Then

$$|\Psi\rangle_1 * |\Psi\rangle_2 \longleftrightarrow \Psi_1^M * \Psi_2^M$$

Witten   Moyal

Sliver takes form

$$|\Xi\rangle = \mathcal{N}^{26}\, e^{-\frac{1}{2}\int_0^\infty d\kappa\, \frac{\theta_\kappa - 2}{\theta_\kappa + 2}\left(e_\kappa^+ e_\kappa^+ + o_\kappa^+ o_\kappa^+\right)}|0\rangle$$

MOYAL REPRESENTATION OF SFT

AIM: Writing VSFT in terms of Moyal $*$ product

First, define

$$o_\kappa^+ = -\sqrt{2}\, i \sum_{n=1}^{\infty} v_{2n-1}(\kappa)\, a_{2n-1}^+$$

$$e_\kappa^+ = \sqrt{2} \sum_{n=1}^{\infty} v_{2n}(\kappa)\, a_{2n}^+$$

with inverses

$$a_{2n-1}^+ = \sqrt{2}\, i \int_0^\infty d\kappa\, v_{2n-1}(\kappa)\, o_\kappa^+$$

$$a_{2n}^+ = \sqrt{2} \int_0^\infty d\kappa\, v_{2n}(\kappa)\, e_\kappa^+$$

and commutators

$$[o_\kappa, o_{\kappa'}^+] = [e_\kappa, e_{\kappa'}^+] = \delta(\kappa-\kappa'), \qquad [o_\kappa, e_{\kappa'}^+] = [e_\kappa, o_{\kappa'}^+] = 0$$

The 3-strings vertex becomes

$$|V_3\rangle = \exp\Big[\int_0^\infty d\kappa \Big\{-\frac{1}{2}\mu(\kappa)\left(o_\kappa^{(1)+} o_\kappa^{(1)+} + e_\kappa^{(1)+} e_\kappa^{(1)+} + \text{cyc.}\right)$$
$$-\frac{1}{2}\left(\mu^{12}(\kappa)+\mu^{21}(\kappa)\right)\left(o_\kappa^{(1)+} o_\kappa^{(2)+} + e_\kappa^{(1)+} e_\kappa^{(2)+} + \text{cyc.}\right)$$
$$-\frac{i}{2}\left(\mu^{12}(\kappa)-\mu^{21}(\kappa)\right)\left(e_\kappa^{(1)+} o_\kappa^{(2)+} - o_\kappa^{(1)+} e_\kappa^{(2)+} + \text{cyc.}\right)\Big\}\Big]|0\rangle$$

Spectrum (D.M. Belov, B. Feng, Y.H. He, N. Moeller)

The continuous spectrum of X is the same as for X'

$$-\infty \leq k \leq +\infty$$

$$-\frac{1}{3} \leq \mu(k) = X(k) \leq 0$$

$$-1 \leq t(k) = -e^{-\frac{\pi |k|}{2}} \leq 0$$

In addition X' has discrete spectrum

$$X' = \frac{1}{3}\left(1 + CU' + U'C\right)$$

CU' is unitary (eigenvalues in the unit circle).

Let us denote the discrete eigenvalues by ξ and $\bar{\xi}$

Eigenvectors: $V_N^{(k)}$ $-\infty \leq k \leq +\infty$, $V_N^{(\xi)}$, $V_N^{(\bar{\xi})}$

Completeness:

$$\int_{-\infty}^{+\infty} dk\, V_N^{(k)} V_M^{(k)} + V_N^{(\xi)} V_M^{(\xi)} + V_N^{(\bar{\xi})} V_M^{(\bar{\xi})} = \delta_{NM}$$

For instance:

$$a_k = \sum_{N=0}^{\infty} V_N^{(k)} a_N \,, \qquad a_\xi = \sum_{N=0}^{\infty} V_N^{(\xi)} a_N \,, \qquad a_{\bar{\xi}} = \sum_{N=0}^{\infty} V_N^{(\bar{\xi})} a_N$$

$$a^+ S' a^+ = \frac{1}{2}\int_{-\infty}^{+\infty} dk\; t(k)\left(e_k^+ e_k^+ + o_k^+ o_k^+\right) + \underbrace{t(\eta)}_{e^{-|\eta|}}\left(e_\eta^+ e_\eta^+ + o_\eta^+ o_\eta^+\right)$$

$$e_k = \frac{a_k + C a_k}{\sqrt{2}} \qquad e_\eta = \frac{a_\xi + C a_\xi}{\sqrt{2}} \qquad C a_k = a_{-k}$$

$$o_k = \frac{a_k - C a_k}{\sqrt{2}} \qquad o_\eta = \frac{a_\xi - C a_\xi}{\sqrt{2}} \qquad C a_\xi = a_{-\xi}$$

where

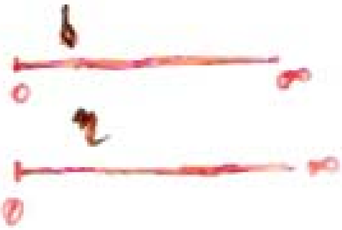
$$\xi = -\frac{2 - \cosh\eta - i\sqrt{3}\,\sinh\eta}{1 - 2\cosh\eta}$$

$$F(\eta) = \psi\left(\tfrac{1}{2} + \tfrac{\eta}{2\pi i}\right) - \psi\left(\tfrac{1}{2}\right) \qquad \psi(z) = \frac{d \log\Gamma(z)}{dz}$$

$$\mathrm{Re}\, F(\eta) = \frac{b}{4}$$

For any b there corresponds a couple $\eta, -\eta$

$b \approx 0,\ \eta \approx 0$



$b \sim \infty \quad b \simeq 4\log\eta$

Conclusion :

$$s' \equiv S'_{00} = \int_{-\infty}^{+\infty} dk\, V_0^{(k)} \left(-e^{-\pi\frac{|k|}{2}}\right) V_0^{(k)} + V_0^{(\xi)} e^{-|\eta|} V_0^{(\xi)} + V_0^{(\bar\xi)} e^{-|\eta|} V_0^{(\bar\xi)}$$

$$V_0^{(\xi)} = \frac{1}{\sqrt{\sinh\eta \frac{\partial}{\partial\eta} \log \mathrm{Re}\, F(\eta)}}$$

$V_0^{(\xi)} = \frac{1}{\sqrt{2}} + O(\eta)\,,\quad \eta \simeq 0 \qquad V_0^{(\xi)} \simeq e^{-\frac{\eta}{2}} \sqrt{2\eta\log\eta} \quad \eta \simeq \infty$

True solution.

(L.B., C. Maccaferri, R. Scherer, D. Tolla)

Unconventional ~~sliver~~ lump.

$$\check{T}'_{NM} = \int_{-\infty}^{+\infty} dk\, V_N^{(k)} V_M^{(k)} \left(-e^{\frac{\pi|k|}{2}}\right) + V_N^{(\xi)} V_M^{(\xi)} e^{|\eta|} + V_N^{(\bar\xi)} V_M^{(\bar\xi)} e^{|\eta|}$$

This is still a well-defined projector:

$$|\check\Lambda'\rangle = \mathcal{N}' \, e^{-\frac{1}{2} a^\dagger C \check{T}' a^\dagger} |\Omega_b\rangle$$

For instance

$$\sqrt{\det(1-\check{T}')} = \sqrt{\det(1-\check{T}')^{(d)} \det(1-\check{T}')^{(c)}}$$

but

$$\det(1-\check{T}')^{(d)} = \left(1 - e^{\eta}\right)^2$$

Conventional lump and unconventional lump have the same energy.

Now define

$$|y\rangle = \frac{1}{\sqrt{\pi}} e^{-\frac{1}{2} y^2 - \sqrt{2}\, i\, a_\eta^\dagger y + \frac{1}{2} a_\eta^\dagger a_\eta^\dagger} |\Omega_\eta\rangle$$

$$a_\eta |\Omega_\eta\rangle = 0$$

$$\hat{y}_\eta |y\rangle = y |y\rangle$$

$$|x_0\rangle = \sqrt{\frac{2}{b\pi}}\, e^{-\frac{1}{b} x_0^2 - \frac{1}{\sqrt{b}}\, i\, a_0^\dagger x_0 + \frac{1}{2}(a_0^\dagger)^2} |\Omega_b\rangle$$

$$a_0 |\Omega_b\rangle = 0$$

$$\hat{x}_0 |x_0\rangle = x_0 |x_0\rangle$$

After inverse Wick-rotation:

$$\langle x_0, y | 1' \rangle = \sqrt{\frac{2}{b\pi}} \frac{\mathcal{N}'}{\sqrt{2\pi(1+e^{\eta})}} e^{\frac{1-e^{\eta}}{1+e^{\eta}} y^2} \frac{1}{\sqrt{1+\check{T}'_{00}}} \cdot$$

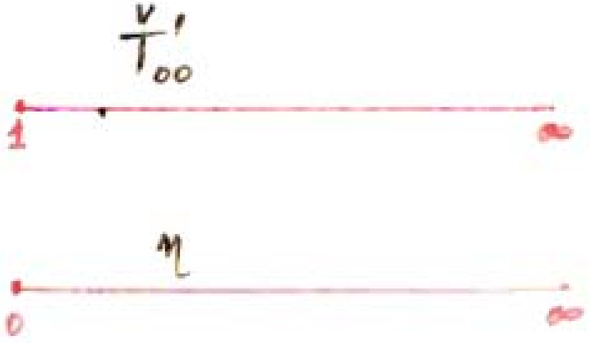
$$\cdot\, e^{\frac{1}{b}\frac{1-\check{T}'_{00}}{1+\check{T}'_{00}} x_0^2 + \frac{2i}{\sqrt{b}(1+\check{T}'_{00})} x_0 \check{T}'_{0n} a_n^+ - \frac{1}{2} a_n^+ W''_{nm} a_m^+} |0\rangle$$

where

$$W''_{nm} = \check{S}''_{nm} - \frac{1}{1+\check{T}'_{00}} \check{S}'_{0n} \check{S}'_{0m}$$

$$\check{S}''_{nm} = \left((-1)^n + (-1)^m\right)\left(-\int_0^\infty V_n^{(k)} V_m^{(k)} e^{-\frac{\pi k}{2}} + V_n^{(\eta)} V_m^{(\eta)} e^{|\eta|}\right) \quad n, m \text{ even}$$

$$= -\left((-1)^n + (-1)^m\right) \int_0^\infty V_n^{(k)} V_m^{(k)} e^{-\frac{\pi k}{2}} \quad n, m \text{ odd}$$



Therefore

$$\boxed{0 \geq \frac{1-\check{T}'_{00}}{1+\check{T}'_{00}} \geq -1} \qquad -1 \leq \frac{1-\check{T}'_{00}}{1+\check{T}'_{00}} \leq 0$$

When $b \to \infty$ static solution = initial sliver

So $\frac{1}{b} \sim \lambda_{sen}$

Low energy limit $\alpha' \to 0$ $\quad \alpha' = \text{length}^2 \cdot \epsilon$

$$W''_{nm} = \delta_{nm} + O\left(\frac{\epsilon}{b}\right)$$

$$\frac{1-\check{T}'_{00}}{1+\check{T}'_{00}} \to -1 \qquad \epsilon \to 0$$

$$\frac{\check{T}'_{0n}}{1+\check{T}'_{00}} \to 0 \qquad \epsilon \to 0$$

$$\langle x_0, y | \Lambda' \rangle \to \mathcal{N}' e^{-\frac{x_0^2}{b}} |\Xi\rangle$$

Tensionless limit: (extremely high energy limit)

$$\lim_{\eta \to 0} \frac{1-\check{T}'_{00}}{1+\check{T}'_{00}} = 0$$

Every mode becomes massless: no decay!

Limit $y \to \infty$:

$$\langle x_0, y | \Lambda' \rangle \longrightarrow 0 \qquad \text{stable vacuum}$$

What is "tachyon matter"? (Gaiotto, Itzhaki, Rastelli)

The marginal deformation

$$\tilde{\lambda} \int_{\partial\Sigma} dt \, \cosh X^0(t)$$

seems to represent, for $\tilde{\lambda} = \frac{1}{2}$, the tachyon vacuum.

It is equivalent to an infinite array of D-branes at

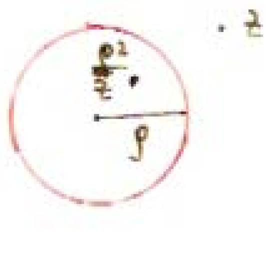
$$X^0 = i\,(2n+1)\pi a$$

Relation between Wick-rotated amplitudes $\hat{A}(E)$ and inverse-Wick-rotated amplitudes $S(E)$

$$S(E) = \frac{1}{2\sinh\frac{aE}{2}} \, \mathrm{Disc}_E\left[\hat{A}(iE)\right]$$

$$\mathrm{Disc}_E\, f(E) = \frac{1}{i}\Big(f(E+i\epsilon) - f(E-i\epsilon)\Big)$$

For instance, 2-point closed string tachyon on the disk $= \{ z, |z| \geq \rho \}$



$$\tilde{A}(p_1, p_2) = \int_0^1 d\rho \; \rho^{t/2-3} \left(1-\rho^2\right)^{s-2} = \frac{\Gamma(t/4-1)\,\Gamma(s-1)}{2\,\Gamma\left(\frac{t}{4}+s-2\right)}$$

Open string channel $\quad s = 1, 0, -1, \ldots$

Closed string channel $\quad t = 4, 0, -4, \ldots$

$$S(p_1, p_2) = \frac{1}{2\sinh\frac{a|E|}{2}} \sum_{k=0}^{\infty} f_k(s)\, \delta\left(\frac{t}{4} - 1 + k\right)$$

- S has poles in t, not in s (no open string poles)
- contributions to S come from $\rho \simeq 0$ (disk $\rightarrow$ sphere)

Conclusion $\quad S$ describes a sphere amplitude with two tachyon insertions and a tower of on-shell massive closed string states.

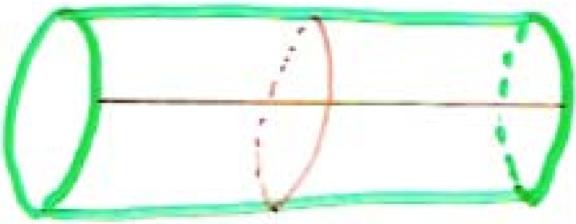
BCFT description

$$|w\rangle \equiv \frac{\delta(L_0 + \bar{L}_0)}{2\sinh\frac{a|E|}{2}} \left(b_0 + \bar{b}_0\right) |B^{p-1}\rangle_{|z|=1}$$

$$|B^{p-1}\rangle = N\, \delta(x^\mu)\; e^{+a_m^{\mu+}\eta_{\mu\nu}\tilde{a}_m^{\nu+} - a_m^{m+}\delta_{mp}\tilde{a}_m^{p+}} |0\rangle$$

At $a = 2\pi$ ($\tilde{\lambda} = \frac{1}{2}$) the energy is stored in very massive closed string modes, which behave non-relativistically and are localized near the brane $\Longrightarrow$ tachyon matter

There seems to be an open-closed string duality at tree level.



traditional duality

"Tree level open strings know about closed strings"

OS

CS

$$|\Xi\rangle = \mathcal{N}\, e^{-\frac{1}{2} a^{+} S a^{+}} |0\rangle$$

$$a^{+} S a^{+} = \sum_{n,m=1}^{\infty} a_n^{+} S_{nm} a_m^{+}$$

$$|B\rangle \sim e^{-\sum_{n=1}^{\infty} a_n^{+} \tilde{a}_n^{+}} |0\rangle$$

D-branes

Tachyon driven cosmology

$$S = -\frac{1}{16\pi G}\int d^4x\left[-\sqrt{g}\,R + V(T)\sqrt{\det\left(g+\partial T\partial T\right)} + \Lambda\sqrt{g}\right]$$

with

$$V(T) = \frac{V_0}{\cosh\frac{T}{\sqrt{2}}}$$

Friedmann eqs.

$$H^2 = \frac{8\pi G}{3}\left[\frac{V(T)}{\sqrt{1-\dot{T}^2}} + \Lambda\right]$$

$$\frac{\ddot{a}}{a} = \frac{8\pi G}{3}\left[\Lambda + \frac{V(T)}{\sqrt{1-\dot{T}^2}} - \frac{3}{2}\,\frac{\dot{T}^2 V(T)}{\sqrt{1-\dot{T}^2}}\right]$$

$$\ddot{T} = -\left(1-\dot{T}^2\right)\left[\frac{V'(T)}{V(T)} + 3\dot{T}\,\frac{\dot{a}}{a}\right]$$

Not good inflation $\eta \sim 1$

Perhaps good model for dark matter

or pre-inflationary cosmology.

But OSFT potential not $V(T)$!