Bosonic Open String Field Theory (D=26)

Action

$$S = -\frac{1}{g_0^2}\left(\frac{1}{2}\int \Psi * Q_B \Psi + \frac{1}{3}\int \Psi * \Psi * \Psi\right)$$

where

$$Q_B^2 = 0$$

$$\int Q_B \Psi = 0$$

$$(A * B) * C = A * (B * C)$$

$$Q_B(A * B) = (Q_B A) * B + (-1)^{|A|} A * (Q_B B)$$

Gauge invariance:

$$\delta\Psi = Q_B \Lambda + \Psi * \Lambda - \Lambda * \Psi$$

By definition $\quad |A| = $ Grassmannality of A

$$\#_g(\Psi) = \#_g(Q_B) = 1$$

$$\#_g(\Lambda) = 0$$

$$\#_g(*) = 0$$

$$\#_g\left(\int\right) = -3$$

Definitions:

1) The vacuum ($SL(2,\mathbb{R})$ invariant)

$$\alpha_n^\mu |0\rangle = 0 \qquad n \geq 0$$

$$c_n |0\rangle = 0 \qquad n > 1$$

$$b_n |0\rangle = 0 \qquad n \geq -1$$

2) The string Field

$$\Psi[x(\sigma)]$$

or

$$|\Psi\rangle = \left(\phi(x) + A_\mu(x)\,\alpha_{-1}^\mu + B_{\mu\nu}(x)\,\alpha_{-1}^\mu \alpha_{-1}^\nu + \ldots\right) c_1 |0\rangle$$

Relation between the two: define

$$a_n^\mu = \frac{1}{\sqrt{n}} \alpha_n^\mu \qquad\qquad a_n^{\mu +} = \frac{1}{\sqrt{n}} \alpha_{-n}^\mu$$

$$\hat{x}_n = \frac{i}{\sqrt{2n}} \left(a_n - a_n^+\right) \qquad\qquad \hat{p}_n = \sqrt{\frac{n}{2}} \left(a_n + a_n^+\right)$$

$$\hat{x}(\sigma) = \hat{x}(\sigma, \tau = 0) = \hat{x}_0 + \sqrt{2} \sum_{n=1}^{\infty} \hat{x}_n \cos n\sigma$$

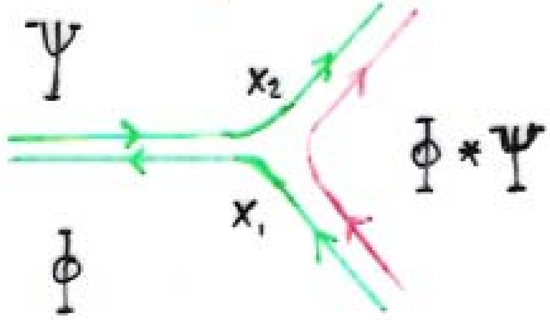
then

$$\varphi[x(\sigma)] = \langle \hat{x}(\sigma) | \Psi \rangle$$

$$|\hat{x}(\sigma)\rangle = \exp \sum_{n=1}^{\infty} \left(-\frac{1}{2} n\, x_n x_n - x_0^2 - i\sqrt{2n}\, a_n^+ x_n - 2i\, a_0^+ x_0 + \frac{1}{2} a_n^+ a_n^+ \right) |0\rangle$$

3) The $*$ product.

Star product of $\Phi[x_1]$ with $\Psi[x_2]$ means identifying R half of x_1 with L half of x_2 and integrating over



- First formulation (functional)

$$(\Phi * \Psi)[z(\sigma)] = \int \Phi[x(\sigma)]\, \Psi[y(\sigma)] \prod_{\frac{\pi}{2}\leq\sigma\leq\pi} \delta[x(\sigma)-y(\pi-\sigma)] \prod_{\frac{\pi}{2}\leq\sigma\leq\pi} dx(\sigma) \prod_{0\leq\sigma\leq\frac{\pi}{2}} dy(\sigma)$$

$$z(\sigma) = x(\sigma) \qquad 0\leq\sigma\leq\frac{\pi}{2}$$

$$z(\sigma) = y(\sigma) \qquad \frac{\pi}{2}\leq\sigma\leq\pi$$

- Second formulation (operator)

3-string vertex $\langle V_3|$

$$\boxed{\begin{aligned}\langle V_3| = {}_1\langle 0|\, c_{-1}^{(1)} c_0^{(1)} \otimes {}_2\langle 0|\, c_{-1}^{(2)} c_0^{(2)} \otimes {}_3\langle 0|\, c_{-1}^{(3)} c_0^{(3)} \cdot \int dp_1\, dp_2\, dp_3\, \delta(p_1+p_2+p_3) \\ \cdot \exp\left[-\left(\frac{1}{2}\sum_{r,s=1}^{3}\sum_{n,m=0}^{\infty} \eta_{\mu\nu}\, a_n^{(r)\mu} V_{nm}^{rs} a_m^{(s)\nu} + \sum_{r,s=1}^{3}\sum_{\substack{n\geq 1\\ m\geq 0}} c_n^{(r)} \tilde{V}_{nm}^{rs} b_m^{(s)}\right)\right]\end{aligned}}$$

where $$\left[a_m^{(r)\mu}, a_n^{(s)\nu\dagger}\right] = \eta^{\mu\nu}\delta_{mn}\delta^{rs}$$

$a_n = \frac{\alpha_n}{\sqrt{n}}$

$$\hat{p}|p\rangle = p|p\rangle \quad , \quad \langle p|p'\rangle = \delta(p+p')$$

Then

$$\boxed{{}_3\langle \phi * \psi| = \langle V_3|\phi\rangle_1 |\psi\rangle_2}$$

where

$$\langle\phi| = bpz(|\phi\rangle)$$

Rules for bpz:

$$bpz(\alpha^\mu_{-m}) = -(-1)^m \alpha^\mu_m$$

$$bpz(c_{-m}) = -(-1)^m c_m$$

$$bpz(b_{-m}) = (-1)^m b_m$$

Use:

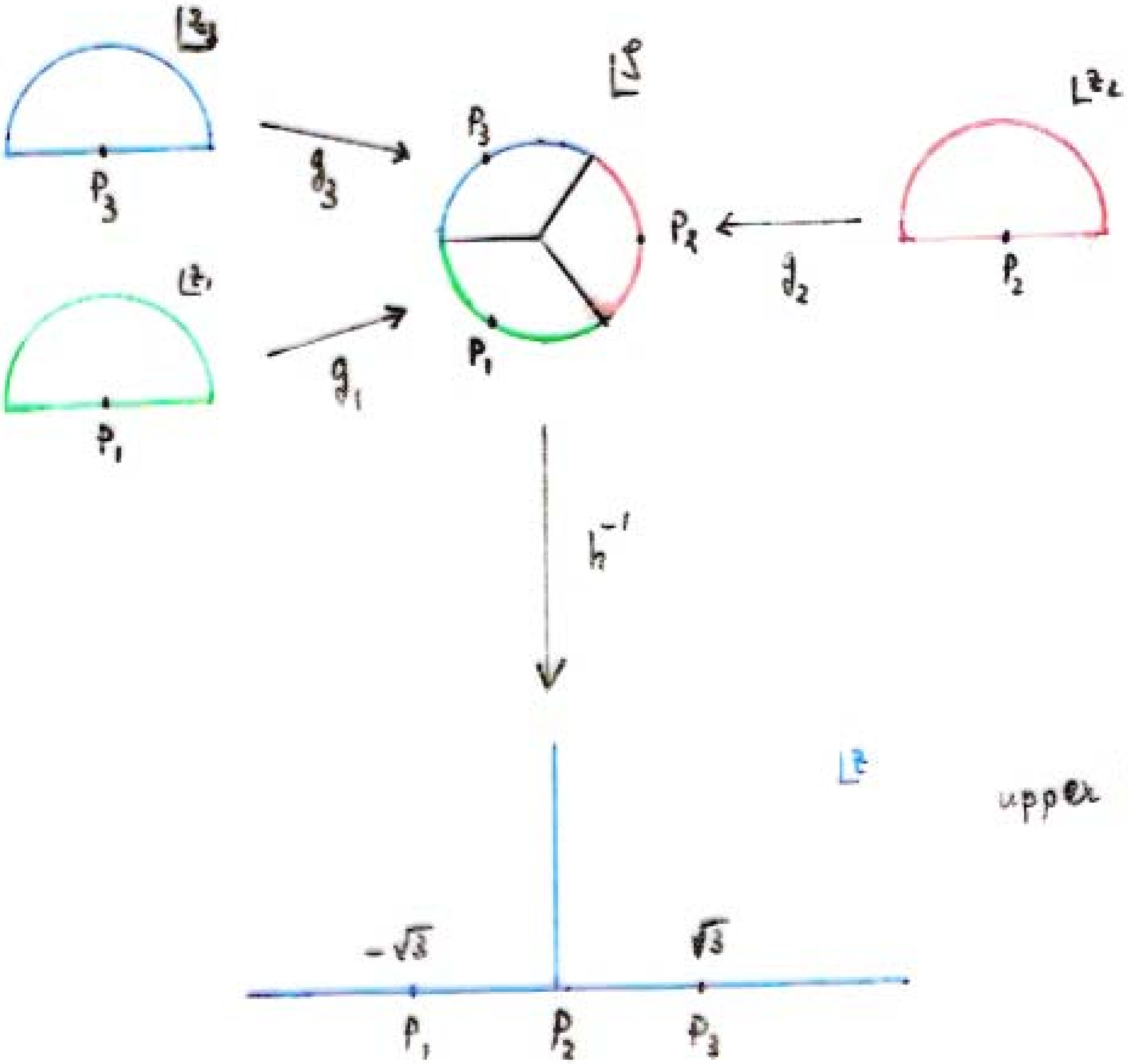
$$\langle 0| e^{\lambda_i a_i - \frac{1}{2} a_i P_{ij} a_j} \; e^{\mu_i a_i^\dagger - \frac{1}{2} a_i^\dagger Q_{ij} a_j^\dagger} |0\rangle =$$

$$= (\det K)^{-1/2} \; e^{\mu^T K^{-1}\lambda - \frac{1}{2}\lambda^T Q K^{-1}\lambda - \frac{1}{2}\mu^T K^{-1} P \mu}$$

with $$K = 1 - PQ$$

Third formulation

CFT formulation



upper half-plane H

$$g_k(z_k) = e^{\frac{2\pi i}{3}(k-2)} \left(\frac{1+iz_k}{1-iz_k}\right)^{2/3} \qquad z = h^{-1}(\xi) = -i\,\frac{\xi-1}{\xi+1}$$

$$f_k(z_k) = h^{-1}\circ g_k(z_k)$$

Then

$$\int \Phi * \Phi * \Phi = \langle f_1\circ\Phi(0)\;\, f_2\circ\Phi(0)\;\, f_3\circ\Phi(0)\rangle_H$$

4) The BRST charge

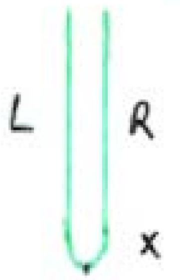
$$\boxed{Q_B = \sum_{m=-\infty}^{+\infty} c_m L^{(m)}_{-m} + \sum_{m,k} \frac{m-k}{2} : c_m c_k b_{-m-k} : - c_0}$$

$$Q_B^2 = 0 \quad \text{in } D = 26$$

$\{Q_B, b_0\} = L_0^{tot} \rightarrow$ Siegel gauge $b_0|\psi\rangle = 0$

5) Integration

Integration corresponds to identifying L and R of string and integrating over



$$\Leftrightarrow \quad \int \Phi[x] = \langle I | \Phi \rangle$$

where

$$I[x(\sigma)] = \langle x(\sigma) | I \rangle = \prod_{0 \le \sigma \le \pi/2} \delta\big(x(\sigma) - x(\pi - \sigma)\big)$$

More explicitly

$$\int \Phi \equiv \int \mathcal{D}x(\sigma) \prod_{0 \le \sigma \le \frac{\pi}{2}} \delta\big(x(\sigma) - x(\pi-\sigma)\big)\, \Phi[x(\sigma)]$$

In operator language $\langle I | = \langle I_m | \otimes \langle I_g |$:

$$\langle I_m | = \langle 0 | e^{-\frac{1}{2}\sum_n a_n C_{nm} a_m} \qquad C_{nm} = (-1)^n \delta_{nm}$$

$$\langle I_g | = \langle 0 | e^{-\sum_{n=1}^{\infty} (-1)^n c_n b_n}$$

Some examples

- $|I\rangle$ is the identity for the $*$ product

$$\left(\Phi * I\right)[z(\sigma)] = \int \Phi[x(\sigma)] \prod_{0\le\sigma\le\frac{\pi}{2}} \delta(y(\sigma)-y(\pi-\sigma)) \cdot$$

$$\cdot \prod_{\frac{\pi}{2}\le\sigma\le\pi} \delta[x(\sigma)-y(\pi-\sigma)] \prod_{\frac{\pi}{2}\le\sigma\le\pi} dx(\sigma) \prod_{0\le\sigma\le\frac{\pi}{2}} dy(\sigma)$$

$$= \int \Phi[x(\sigma)] \prod_{\frac{\pi}{2}\le\sigma\le\pi} \delta[x(\sigma)-y(\sigma)] \prod_{\frac{\pi}{2}\le\sigma\le\pi} dx(\sigma)$$

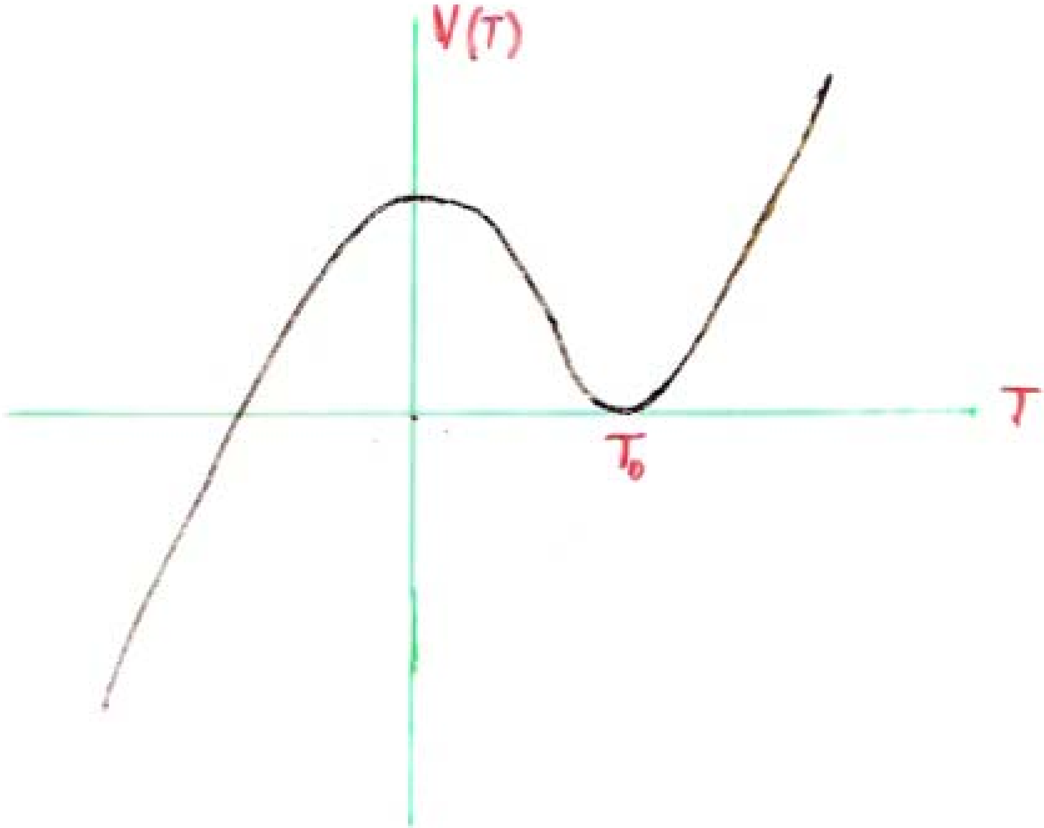
$$= \Phi[y(\sigma)] \quad \tfrac{\pi}{2}\le\sigma\le\pi \qquad = \Phi[x(\sigma)] \quad 0\le\sigma\le\tfrac{\pi}{2}$$

$$= \Phi[z(\sigma)]$$

Another representation of $|I\rangle$:

$$|I\rangle = e^{L_{-2}-\frac{1}{2}L_{-4}+\frac{1}{2}L_{-6}-\frac{7}{12}L_{-8}\cdots}|0\rangle$$

Sen's conjectures (on D=26 OBS)



$$V(T) = M\left(1 + f(T)\right) \qquad M = T_{25}$$

1) $f(T_0) = -1$

2) There exist soliton lumps that correspond to lower dimensional branes

3) The vacuum at T_0 is the closed string vacuum

If we restrict Φ to

$$|\Phi\rangle = \int d^d k \left(\phi(k) + A_\mu(k)\,\alpha_{-1}^\mu\right) c_1 |k\rangle$$

the action becomes (Siegel gauge $b_0|\Phi\rangle = 0$)

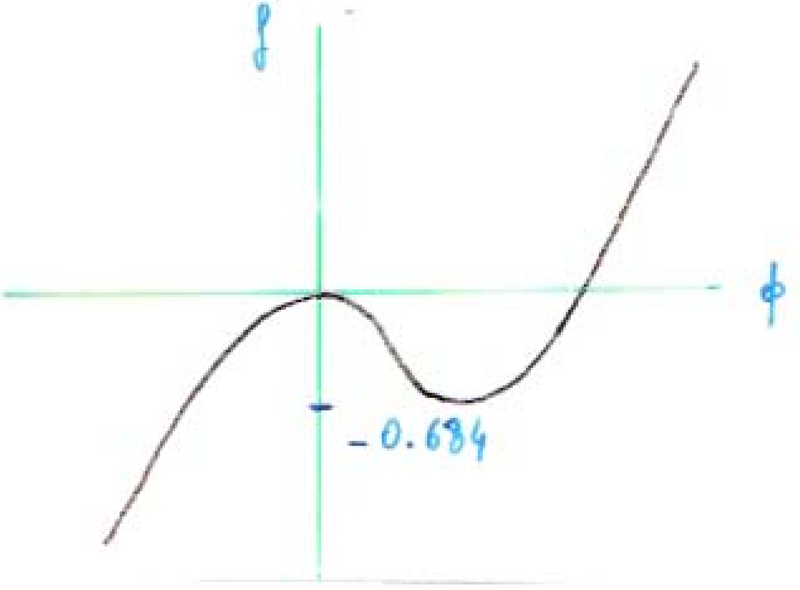
$$\begin{aligned} S = \frac{1}{g_0^2} \int d^d x \Big(& -\frac{1}{2}\partial_\mu \phi\, \partial^\mu \phi + \frac{1}{2\alpha'}\phi^2 - \frac{1}{2}\partial_\mu A_\nu\, \partial^\mu A^\nu \\ & - \frac{1}{3}\left(\frac{3\sqrt{3}}{4}\right)^3 \tilde{\phi}^3(x) - \frac{3\sqrt{3}}{4}\, \tilde{\phi}\, \tilde{A}_\mu \tilde{A}^\mu + \\ & - \frac{3\sqrt{3}}{8}\alpha' \left(\partial_\mu \partial_\nu \tilde{\phi}\, \tilde{A}^\mu \tilde{A}^\nu + \tilde{\phi}\, \partial_\mu \tilde{A}^\nu \partial_\nu \tilde{A}^\mu - 2\,\partial_\mu \tilde{\phi}\, \partial_\nu \tilde{A}^\mu \tilde{A}^\nu\right)\Big) \end{aligned}$$

where

$$\tilde{f}(x) = e^{-\alpha' \ln \frac{4}{3\sqrt{3}}\, \partial_\mu \partial^\mu} f(x)$$

Considering only the tachyon and dropping derivatives

$$S \longrightarrow \frac{1}{g_0^2} \int d^d x \left(\frac{1}{2\alpha'}\phi^2 - \frac{1}{3}\left(\frac{3\sqrt{3}}{4}\right)^3 \phi^3 \right) \equiv -\frac{f(\phi)V}{2\pi^2\alpha'^3}$$



Level truncation

level	$f(T_0)$
(0,0)	-0.684
(2,4)	-0.949
(2,6)	-0.959
(4,8)	-0.986
(4,12)	-0.988
(6,12)	-0.99514
(6,18)	-0.99518
(8,16)	-0.99777
(8,20)	-0.99793
(10,20)	-0.99912

Level (2.6)

$$|T\rangle = \left(\phi\, c_1 - \beta_1\, c_{-1} + \frac{v}{\sqrt{13}}\, L^{(m)}_{-2}\, c_1\right)|0\rangle$$

gives

$$\begin{aligned} f(T) = 2\pi^2\alpha'^3\Big(& -\frac{1}{2\alpha'}\phi^2 + \frac{3^3\sqrt{3}}{2^6}\phi^3 - \frac{1}{2\alpha'}\beta_1^2 + \frac{1}{2\alpha'}v^2 \\ & - \frac{11\cdot 3\sqrt{3}}{2^6}\phi^2\beta_1 - \frac{5\cdot 3\sqrt{39}}{2^6}\phi^2 v + \frac{19\sqrt{3}}{3\cdot 2^6}\phi\beta_1^2 \\ & + \frac{581\sqrt{3}}{3^2\cdot 2^6}\phi v^2 + \frac{5\cdot 11\sqrt{39}}{3^2\cdot 2^5}\phi\beta_1 v - \frac{1}{2^6\sqrt{3}}\beta_1^3 \\ & - \frac{5\cdot 19\sqrt{39}}{2^6\cdot 3^4}v\beta_1^2 - \frac{6391\sqrt{3}}{2^6\cdot 3^5}v^2\beta_1 - \frac{20951\sqrt{39}}{2^6\cdot 3^5\cdot 13}v^3\Big) \end{aligned}$$

New vacuum has

- no tachyon
- no massless vector field
- $\frac{1}{g_{\text{eff}}} = \frac{V(T)}{g_0}$
- . . . . .

Neumann coefficients

$$\left(\frac{1+ix}{1-ix}\right)^{1/3} = \sum_{n\ \text{even}} A_n x^n + i \sum_{n\ \text{odd}} A_n x^n$$

$$\left(\frac{1+ix}{1-ix}\right)^{2/3} = \sum_{n\ \text{even}} B_n x^n + i \sum_{n\ \text{odd}} B_n x^n$$

$$N^{r,\pm r}_{nm} = \begin{cases} \frac{1}{3(n\pm m)} (-1)^n \left(A_n B_m \pm B_n A_m\right) & m+n \text{ even } n\neq m \\ 0 & m+n \text{ odd} \end{cases}$$

$$N^{r,\pm(r+1)}_{nm} = \begin{cases} \frac{1}{6(n\pm m)} (-1)^{n+1} \left(A_n B_m \pm B_n A_m\right) & m+n \text{ even } n\neq m \\ \frac{1}{6(n\pm m)} \sqrt{3} \left(A_n B_m \mp B_n A_m\right) & m+n \text{ odd} \end{cases}$$

$$N^{r,\pm(r-1)}_{nm} = \begin{cases} \frac{1}{6(n\pm m)} (-1)^{n+1} \left(A_n B_m \mp B_n A_m\right) & m+n \text{ even } m\neq n \\ -\frac{1}{6(n\mp m)} \sqrt{3} \left(A_n B_m \pm B_n A_m\right) & m+n \text{ odd} \end{cases}$$

$$V^{rs}_{nm} = -\sqrt{nm}\left(N^{rs}_{nm} + N^{r,-s}_{nm}\right) \qquad m\neq n,\quad m,n\neq 0$$

$$V^{rr}_{nn} = -\frac{1}{3}\left(2\sum_{k=0}^{n}(-1)^{n-k} A_k^2 - (-1)^n - A_n^2\right) \qquad n\neq 0$$

$$V^{r\,r+1}_{nn} = V^{r\,r+2}_{nn} = \frac{1}{2}\left((-1)^n - V^{rr}_{nn}\right) \qquad n\neq 0$$

$$V^{rs}_{0n} = -\sqrt{2n}\left(N^{rs}_{0n} + N^{r,-s}_{0n}\right) \qquad n\neq 0$$

$$V^{rr}_{00} = \ln\frac{27}{16}$$

Matter Neumann coefficients

$$N^{rs}_{nm} = \langle V_{123} | \alpha^{(r)}_{-n} \alpha^{(s)}_{-m} | 0 \rangle_{123} = \langle f_r[\alpha_{-n}] \, f_s[\alpha_{-m}] \rangle =$$

$$= -\frac{1}{nm} \oint \frac{dz}{2\pi i} \oint \frac{dw}{2\pi i} \frac{1}{z^n} \frac{1}{w^m} f_r'(z) \frac{1}{(f_r(z) - f_s(w))^2} f_s'(w)$$

so that

$$V^{rs}_{nm} = (-1)^{n+m} \sqrt{nm} \, N^{rs}_{nm}$$

Decomposition

$$N^{rs}_{nm} = \frac{1}{3\sqrt{nm}} \left(E_{nm} + \bar{\alpha}^{s-r} U_{nm} + \alpha^{s-r} \bar{U}_{nm} \right) \qquad \alpha = e^{\frac{2\pi i}{3}}$$

where

$$E_{nm} = \frac{-1}{\sqrt{nm}} \oint \frac{dz}{2\pi i} \oint \frac{dw}{2\pi i} \frac{1}{z^n} \frac{1}{w^m} \left(\frac{1}{(1+zw)^2} + \frac{1}{(z-w)^2} \right) = (-1)^n \delta_{nm}$$

$$U_{nm} = \frac{-1}{3\sqrt{nm}} \oint \frac{dz}{2\pi i} \oint \frac{dw}{2\pi i} \frac{1}{z^n} \frac{1}{w^m} \left(\frac{f^2(w)}{f^2(z)} + 2 \frac{f(z)}{f(w)} \right) \left(\frac{1}{(1+zw)^2} + \frac{1}{(z-w)^2} \right)$$

$$\bar{U}_{nm} = (-1)^{n+m} U_{nm}$$

Properties

$$N^{rs}_{nm} = N^{sr}_{mn}$$

$$N^{rs}_{nm} = (-1)^{n+m} N^{sr}_{nm}$$

$$N^{rs}_{nm} = N^{r+1, s+1}_{nm}$$

Integrable structures in SFT

Hirota equations for dTL (dispersionless Toda lattice hierarchy):

H1: $(z_1 - z_2)\, e^{D(z_1) D(z_2) F} = z_1\, e^{-\partial_{t_0} D(z_1) F} - z_2\, e^{-\partial_{t_0} D(z_2) F}$

H2: $z_1 \bar{z}_2 \left(1 - e^{-D(z_1) \bar{D}(\bar{z}_2) F}\right) = e^{\partial_{t_0}(\partial_{t_0} + D(z_1) + \bar{D}(\bar{z}_2)) F}$

where

$$D(z) = \sum_{k=1}^{\infty} \frac{1}{k z^k} \frac{\partial}{\partial t_k} \qquad \bar{D}(\bar{z}) = \sum_{k=1}^{\infty} \frac{1}{k \bar{z}^k} \frac{\partial}{\partial \bar{t}_k}$$

and

$$F = \ln \tau$$

is the free energy of the system.

A. Boyarsky, O. Ruchayskiy: hepth/0211010

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H1 :

$$F_{t_1t_1} = \frac{1}{2} F_{t_0t_2} - \frac{1}{2}\left(F_{t_0t_1}\right)^2$$

$$\frac{1}{2} F_{t_1t_2} = \frac{1}{3} F_{t_0t_3} - \frac{1}{2} F_{t_0t_1} F_{t_0t_2} + \frac{1}{6}\left(F_{t_0t_1}\right)^3$$

$$\frac{1}{4} F_{t_2t_2} + \frac{1}{2}\left(F_{t_1t_1}\right)^2 - \frac{1}{3} F_{t_1t_3} = 0$$

$$\frac{1}{3} F_{t_1t_3} = \frac{1}{4} F_{t_0t_4} - \frac{1}{3} F_{t_0t_1} F_{t_0t_3} - \frac{1}{8}\left(F_{t_0t_2}\right)^2 + \frac{1}{4} F_{t_0t_1}^2 F_{t_0t_2} - \frac{1}{24} F_{t_0t_1}^4$$

$$\vdots$$

H2 :

$$F_{t_1\bar{t}_1} = e^{F_{t_0t_0}}$$

$$F_{t_2\bar{t}_1} = 2\, e^{F_{t_0t_0}} F_{t_1t_0}$$

$$F_{t_3\bar{t}_1} = \frac{3}{2} e^{F_{t_0t_0}} F_{t_2t_0} + \frac{3}{2} e^{F_{t_0t_0}} F_{t_1t_0}^2$$

$$\frac{1}{4} F_{t_2\bar{t}_2} - \frac{1}{2} F_{t_1\bar{t}_1}^2 = e^{F_{t_0t_0}} F_{t_1t_0} F_{t_0\bar{t}_1}$$

$F_{t_0t_0} = \ln \frac{16}{27}$,

$F_{t_1\bar{t}_1} = \frac{16}{27}$, $\quad F_{t_1\bar{t}_2} = \frac{64}{81\sqrt{3}}$, $\quad F_{t_3\bar{t}_1} = -\frac{16}{3^5}$

Use definition of 3-string vertex (LeClair-Peskin-Preitschopf)

$$N^{23}_{nm} = -\frac{1}{n}\oint_0 \frac{dz}{2\pi i}\frac{1}{z^n} f_2'(z)\,\frac{1}{m}\oint\frac{dw}{2\pi i}\frac{1}{w^m} f_3'(w)\,\frac{1}{(f_2(z)-f_3(w))^2}$$

where

$$f_1(z) = e^{\frac{2\pi i}{3}}\left(\frac{1+iz}{1-iz}\right)^{2/3}$$

$$f_2(z) = \left(\frac{1+iz}{1-iz}\right)^{2/3}$$

$$f_3(z) = e^{-\frac{2\pi i}{3}}\left(\frac{1+iz}{1-iz}\right)^{2/3}$$

and identify

$$F_{t_n t_m} \equiv \frac{\partial^2 F}{\partial t_n \partial t_m} = -\sqrt{nm}\,V^{11}_{nm} = nm\,N^{11}_{nm}$$

$$F_{t_n \bar{t}_m} \equiv \frac{\partial^2 F}{\partial t_n \partial \bar{t}_m} = -\sqrt{nm}\,V^{12}_{nm} = nm\,N^{12}_{nm}$$

$$F_{t_0 \bar{t}_n} \equiv \frac{\partial^2 F}{\partial t_0 \partial \bar{t}_n} = \sqrt{\frac{n}{2}}\left(V^{12}_{0n} - V^{22}_{0n}\right) = n\left(N^{12}_{0n} - N^{12}_{0n}\right)$$

$$F_{t_0 t_n} \equiv \frac{\partial^2 F}{\partial t_0 \partial t_n} = \sqrt{\frac{n}{2}}\left(V^{21}_{0n} - V^{11}_{0n}\right) = n\left(N^{11}_{0n} - N^{21}_{0n}\right)$$

One can prove that these quantities satisfy

the Hirota eqs. H1 – H2.

All classical solutions (such as the sliver) satisfy Hirota eq.

Basic property

$$\sum_{k=1}^{\infty} U_{nk} U_{km} = \delta_{nm}$$

it follows

$$X^{rs} = C V^{rs} \qquad [X^{rs}, X^{r's'}] = 0 \quad \forall\, r, s, r', s'$$

Zero modes

$$N_{0n}^{rs} = -\frac{1}{n} \oint \frac{dz}{2\pi i} \frac{1}{z^n} f_s'(z) \frac{1}{f_r(0) - f_s(z)} = \frac{1}{3}\left(E_n + \bar{\alpha}^{r-s} U_n + \alpha^{r-s} \bar{U}_n\right)$$

where

$$E_n = -\frac{4i}{n} \oint \frac{dz}{2\pi i} \frac{1}{z^n} \frac{1}{1+z^2} \frac{f^3(z)}{1-f^3(z)} = 2\frac{i^n}{n}$$

$$U_n = -\frac{4i}{n} \oint \frac{dz}{2\pi i} \frac{1}{z^n} \frac{1}{1+z^2} \frac{f^2(z)}{1-f^3(z)} = \frac{a_n}{n}$$

$$\bar{U}_n = (-1)^n \frac{a_n}{n}$$

$$\left(\frac{1+iz}{1-iz}\right)^{1/3} = \sum_{n=0}^{\infty} a_n z^n$$

Imposing the gauge fixing condition

$$\sum_{r=1}^{3} N_{0n}^{rs} = 0 \quad \Rightarrow \quad \hat{N}_{0n}^{rs} = N_{0n}^{rs} - \frac{1}{3} E_n$$

$$V_{0n}^{rs} = -\sqrt{2n}\, \hat{N}_{0n}^{rs}$$

Vacuum String Field Theory

Defines a SFT corresponding to closed string vacuum. Just shift

$$\Phi = \Phi_0 + \tilde{\Phi} \qquad\qquad \Phi_0 \text{ corresponds to } T_0$$

Then

$$S(\Phi_0 + \tilde{\Phi}) = -V_{25} T_{25} - \frac{1}{g_0^2} \int \Big[\frac{1}{2} (\Phi_0 + \tilde{\Phi}) * Q (\Phi_0 + \tilde{\Phi}) + \\ + \frac{1}{3} (\Phi_0 + \tilde{\Phi}) * (\Phi_0 + \tilde{\Phi}) * (\Phi_0 + \tilde{\Phi}) \Big]$$

$$= -\frac{1}{g_0^2} \int \Big[\frac{1}{2} \tilde{\Phi} * Q \tilde{\Phi} + \frac{1}{3} \tilde{\Phi} * \tilde{\Phi} * \tilde{\Phi} \Big]$$

where

$$Q \tilde{\Phi} = Q_B \tilde{\Phi} + \frac{1}{2} \left(\Phi_0 * \tilde{\Phi} + \tilde{\Phi} * \Phi_0 \right)$$

Possible field redefinition

$$\tilde{\Phi} = e^K \Psi$$

Summing up <u>we postulate</u> at the closed string vacuum

$$S = -\frac{1}{g_0^2} \int \Big[\frac{1}{2} \Psi * \mathcal{Q} \Psi + \frac{1}{3} \Psi * \Psi * \Psi \Big]$$

The new BRST charge $\mathcal{Q}$ satisfies

$$\mathcal{Q}^2 = 0 \qquad\qquad \mathcal{Q}(\psi * \chi) = \mathcal{Q}\psi * \chi + (-1)^\psi \psi * (\mathcal{Q}\chi)$$

The new BRST charge must satisfy

$$\mathcal{Q}^2 = 0$$

$$\mathcal{Q}(A * B) = (\mathcal{Q}A) * B + (-1)^{|A|} A * (\mathcal{Q}B)$$

$$\langle \mathcal{Q}A, B \rangle = -(-1)^{|A|} A * (\mathcal{Q}B)$$

and

- $\mathcal{Q}$ must have vanishing cohomology
  (no open string states)

- $\mathcal{Q}$ must be universal
  (no dependence on BCFT)

Examples of $\mathcal{Q}$'s:

- $\mathcal{Q} = c_0$

- $\mathcal{Q} \equiv \mathcal{C}_n = c_n + (-1)^n c_{-n}$ $\qquad n = 0, 1, 2, \ldots$

- $\mathcal{Q} \equiv \sum_{n=0}^{\infty} a_n \mathcal{C}_n$.

Proof: define $\mathcal{B}_n = \frac{1}{2}\left(b_n + (-1)^n b_{-n}\right) \longrightarrow \{\mathcal{C}_n, \mathcal{B}_n\} = 1$

Therefore, if $\mathcal{C}_n \psi = 0 \longrightarrow \psi = \mathcal{C}_n(\mathcal{B}_n \psi) = \{\mathcal{C}_n, \mathcal{B}_n\}\psi$

Now search for classical solution of EOM of VSFT

$$\mathcal{Q}\Psi = -\Psi * \Psi$$

Ansatz

$$\Psi = \Psi_m * \Psi_g$$

So EOM splits

$$\mathcal{Q}\Psi_g = -\Psi_g *^g \Psi_g \qquad \Psi_m = \Psi_m *^m \Psi_m$$

and

$$S|_\Psi = -\frac{1}{6g_0^2} \langle \Psi_g | \mathcal{Q}\Psi_g \rangle_g \langle \Psi_m | \Psi_m \rangle_m \equiv K \langle \Psi_m | \Psi_m \rangle_m$$

Method of Kostelecky-Potting

Three string vertex $|V_3\rangle$:

$$|V_3\rangle = \int d^{26}p_{(1)}\, d^{26}p_{(2)}\, d^{26}p_{(3)}\, \delta^{(26)}\left(p_{(1)}+p_{(2)}+p_{(3)}\right) e^{-E} |0,p\rangle_{1,2,3}$$

with

$$E = \frac{1}{2}\sum_{\substack{r,s=1\\ m,n\geq 1}}^{3} \eta_{\mu\nu}\, a_m^{(r)\mu\dagger} V_{mn}^{rs} a_n^{(s)\nu\dagger} + \sum_{\substack{r,s=1\\ n\geq 1}}^{3} \eta_{\mu\nu}\, p_{(r)}^{\mu} V_{0n}^{rs} a_n^{(s)\nu\dagger} +$$

$$+ \frac{1}{2}\sum_{r=1}^{3} \eta_{\mu\nu}\, p_{(r)}^{\mu} V_{00}^{rr} p_{(r)}^{\nu} .$$

and

$$|0,p\rangle_{123} = |p_{(1)}\rangle \otimes |p_{(2)}\rangle \otimes |p_{(3)}\rangle$$

For space-time translational invariant solutions

$$E = \frac{1}{2}\sum_{\substack{r,s=1\\ m,n\geq 1}}^{3} \eta_{\mu\nu}\, a_m^{(r)\mu\dagger} V_{mn}^{rs} a_n^{(s)\nu\dagger}$$

ansatz:

$$|\Psi_m\rangle = \mathcal{N}^{26}\, e^{-\frac{1}{2}\eta_{\mu\nu}\sum_{m,n\geq 1} S_{mn} a_m^{\mu\dagger} a_n^{\nu\dagger}} |0\rangle$$

Now impose

$$|\Psi_m *^m \Psi_m\rangle_3 \equiv {}_1\langle\Psi_m|\,{}_2\langle\Psi_m|V_3\rangle = |\Psi_m\rangle_3$$

Get equation

$$|\Psi_m * \Psi_m\rangle_3 = \mathcal{N}^{52} \det\left[(1-\Sigma \mathcal{V})^{-1/2}\right]^{26} \cdot$$

$$\cdot \exp\left[-\frac{1}{2}\eta_{\mu\nu}\left\{\chi^{\mu T}\frac{1}{1-\Sigma\mathcal{V}}\Sigma\chi^{\nu} + a^{(3)\mu+}\cdot V^{33}\cdot a^{(3)\nu+}\right\}\right]|0\rangle_3$$

where

$$\Sigma = \begin{pmatrix} S & 0 \\ 0 & S \end{pmatrix} \qquad \mathcal{V} = \begin{pmatrix} V^{11} & V^{12} \\ V^{21} & V^{22} \end{pmatrix}$$

$$\chi^{\mu T} = \left(a^{(3)\mu+}V^{31},\ a^{(3)\mu+}V^{32}\right) \qquad \chi^{\mu} = \begin{pmatrix} V^{13}a^{(3)\mu+} \\ V^{23}a^{(3)\mu+} \end{pmatrix}$$

Equating and using $V^{r+1,s+1} = V^{r,s}$ mod 3

$$(*) \qquad S = V^{11} + \left(V^{12}, V^{21}\right)\frac{1}{1-\Sigma\mathcal{V}}\Sigma\begin{pmatrix} V^{21} \\ V^{12} \end{pmatrix}$$

Solve for S. seems hopeless

But . . . define

$$X^{rs} = C\, V^{rs}$$

$$\left[X^{rs}, X^{r's'} \right] = 0$$

$$C_{nm} = (-1)^n \delta_{nm}$$

Set

$$X = X^{11} \qquad\qquad T = CS$$

then (*) becomes

$$(T-1)\left(X T^2 - (1+X) T + X \right) = 0$$

i.e.

$$S = CT \qquad\qquad T = \frac{1}{2X}\left(1 + X - \sqrt{(1+3X)(1-X)} \right)$$

Finally the solution is

$$\boxed{|\Xi\rangle \equiv |\Psi_m\rangle = \left(\det(1-X)\, \det(1+T) \right)^{13} e^{-\frac{1}{2}\sum_{m,n\geq 1} a_m^{+} S_{mn} a_n^{+}} |0\rangle}$$

and

$$\boxed{S|_{\Psi} = K\, \frac{V^{(26)}}{(2\pi)^{26}} \left(\det(1-X)^{3/4} \det(1+3X)^{1/4} \right)^{26}}$$

$|\Psi_m\rangle$ is identified with the D25-brane.

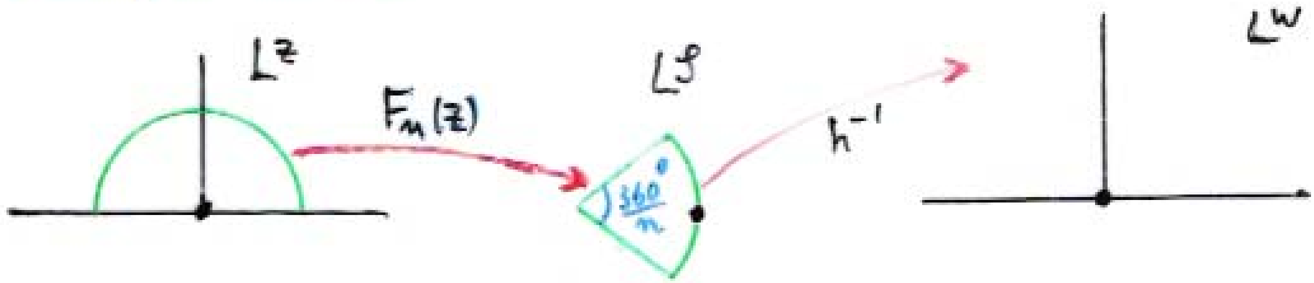
Surface states

defined via conformal map $F(z)$ of the upper half disk to the upper half plane

$\langle f|$ is defined via

$$\langle f|\phi\rangle = \langle f\circ\phi(0)\rangle \qquad |\phi\rangle = \phi(0)|0\rangle$$

Wedge states



$$F_n(z) = \left(\frac{1+iz}{1-iz}\right)^{2/n} \qquad h^{-1}(\xi) = -i\,\frac{\xi-1}{\xi+1}$$

$$f_n = h^{-1}\circ F_n(z) = \mathrm{tg}\left(\frac{2}{n}\,\mathrm{arctg}(z)\right)$$

Then

$$|n\rangle * |m\rangle = |n+m-1\rangle$$

and

$$|n=1\rangle = |I\rangle \qquad |n=\infty\rangle = \text{sliver}$$

Representation of wedge states $|n\rangle$

1) $\langle n|\phi\rangle \equiv \langle f_n \circ \phi(0)\rangle$ for any state $|\phi\rangle = \phi(0)|0\rangle$

$$f_n(z) = \frac{n}{2}\,\mathrm{tg}\left(\frac{2}{n}\,\mathrm{tg}^{-1}(z)\right)$$

2) $$|n\rangle = \exp\left(-\frac{n^2-4}{3n^2}L_{-2} + \frac{n^4-16}{30n^4}L_{-4} - \frac{(n^2-4)(176+128n^2+11n^4)}{1890n^6}L_{-6} + \dots\right)|0\rangle$$

Star product of wedge states

$$|n\rangle * |m\rangle = |n+m-1\rangle$$

Two states satisfy $\psi * \psi = \psi$

$n=1$ <u>identity state</u> $|I\rangle \equiv |1\rangle$

$n=\infty$ <u>sliver state</u> $|\Xi\rangle \equiv |\infty\rangle$

$|I\rangle$

$|\infty\rangle$

Lump solutions

They are supposed to represent D-(25-k)-branes.

k transverse directions, $\alpha = 1, \dots, k$.

Replace

$$|p\rangle = \frac{1}{\pi^{k/4}} e^{-\frac{b}{4} p^\alpha p^\alpha + \sqrt{b}\, a_0^{\alpha +} p^\alpha - \frac{1}{2} a_0^{\alpha +} a_0^{\alpha +}} |\Omega_b\rangle$$

where

$$a_0^\alpha = \frac{1}{\sqrt{2}}\left(\sqrt{b}\,\hat{p}^\alpha - i \frac{\hat{x}^\alpha}{\sqrt{b}}\right) \qquad a_0^{\alpha +} = \frac{1}{\sqrt{2}}\left(\sqrt{b}\,\hat{p}^\alpha + i \frac{\hat{x}^\alpha}{\sqrt{b}}\right)$$

$$[a_0^\alpha , a_0^{\beta +}] = \delta^{\alpha\beta}$$

Integrate over p^α. The relevant vertex is:

$$|V_3\rangle = \exp\left(-\frac{1}{2} \sum_{\substack{r,s \\ m,n \geq 1}} \eta_{\mu\nu} a_m^{(r)\mu +} V_{mn}^{rs} a_n^{(s)\nu +}\right) |0,p\rangle_{123} \cdot$$

$$\cdot \left(\frac{\sqrt{3}}{(16\pi)^{1/4}} \left(V_{00}^{rs} + 1\right)\right)^{-k} \exp\left(-\frac{1}{2} \sum_{\substack{r,s \\ M,N \geq 0}} a_M^{(r)\alpha +} V'^{rs}_{MN} a_N^{(s)\alpha +}\right) |\Omega\rangle$$

$\mu = 0, \dots, 25-k-1$ $\qquad\qquad M = \{0, m\}$

The solution of $|\Psi_{m}\rangle * |\Psi_{m}\rangle = |\Psi_{m}\rangle$ is

$$|\Psi'_{m}\rangle = \left(\sqrt{\det(1-X)\det(1+T)}\right)^{26-k} e^{-\frac{1}{2}\sum_{m,n\geq 1} a_m^{\dagger} S_{mn} a_n^{\dagger}} |0\rangle$$

$$\otimes \left(\frac{\sqrt{3}}{(16\pi)^{1/4}}\left(V_{00}^{22}+1\right)\right)^{k} \left(\det(1-X')\det(1+T')\right)^{k/2} e^{-\frac{1}{2}\sum_{M,N\geq 0} a_M'^{\dagger} S'_{MN} a_N'^{\dagger}} |\Omega\rangle$$

Gives the action

$$S_{\Psi'} = K \frac{V^{(26-k)}}{(2\pi)^{26-k}} \left(\det(1-X)^{3/4}\det(1+3X)^{1/4}\right)^{26-k} \cdot$$

$$\cdot \left(\frac{3}{(16\pi)^{1/2}}\left(V_{00}^{22}+1\right)^2\right)^{k} \left(\det(1-X')^{3/4}\det(1+3X')^{1/4}\right)^{k}$$

Ratio of tensions:

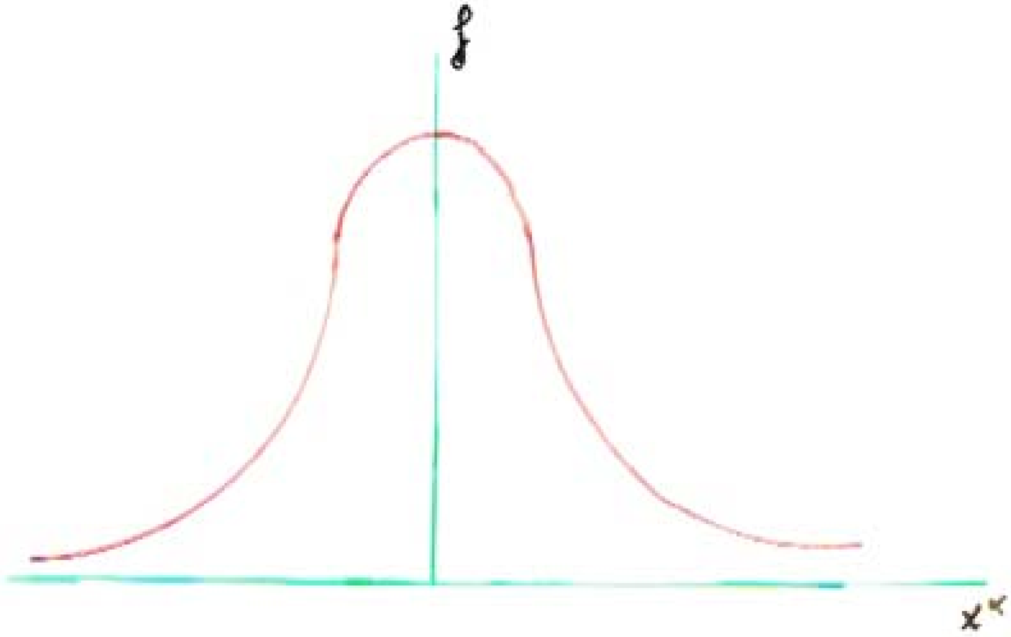
$$\frac{T_{24-k}}{2\pi\alpha' T_{25-k}} = \frac{3}{\sqrt{16\pi}}\left(V_{00}^{22}+1\right)^2 \frac{\det(1-X')^{3/4}\det(1+3X')^{1/4}}{\det(1-X)^{3/4}\det(1+3X)^{1/4}}$$

Numerically this $= 1$. (Okuyama)

In transverse direction α introduce

$$|x^\alpha\rangle \sim e^{-\frac{1}{2}x^\alpha x^\alpha - \frac{2}{\sqrt{b}}\, i\, a_0^{\alpha\dagger} x^\alpha + \frac{1}{2} a_0^{\alpha\dagger} a_0^{\alpha\dagger}}$$

Space profile of a lump solution is : $\langle x^\alpha|\psi'\rangle = f(x^\alpha)\,|\Xi\rangle$



The ghost sector (Hata, Kawano)

After factorization we have to solve

$$\mathcal{Q}\Psi_g + \Psi_g *^g \Psi_g = 0$$

where

$$\mathcal{Q} = c_0 + \sum_{n=1}^{\infty} f_n \mathcal{C}_n \qquad \mathcal{C}_n = c_n + (-1)^n c_{-n}$$

The vacua are

$$|\dot{0}\rangle = c_1|0\rangle \qquad |\hat{0}\rangle = c_0 c_1|0\rangle$$

and the 3-strings vertex

$$|V_3\rangle = e^{\sum_{n,m\geq 1} c_n^{(r)\dagger} \tilde{V}_{nm}^{rs} b_m^{(s)\dagger} + \sum_{n\geq 1} c_n^{(r)\dagger} \tilde{V}_{n0}^{rs} b_0^{(s)\dagger}} |\hat{0}\rangle_1 |\hat{0}\rangle_2 |\hat{0}\rangle_3$$

The ansatz for Ψ_g is

$$|\Psi_g\rangle = b_0|\dot{\Phi}_g\rangle \qquad (\text{Siegel gauge: } b_0|\Psi_g\rangle = 0)$$

$$|\dot{\Phi}_g\rangle = \mathcal{N}_g \, e^{\sum_{n,m\geq 1} c_n^\dagger \tilde{S}_{nm} b_m^\dagger} |\dot{0}\rangle$$

One finds that

$$\tilde{T} = C\tilde{S} \qquad \tilde{T} = \frac{1}{2\tilde{X}}\left(1 + \tilde{X} - \sqrt{(1+3\tilde{X})(1-\tilde{X})}\right)$$

$$\vec{f} = \frac{1}{1-\tilde{T}}\left[\vec{y} + (\tilde{X}_+, \tilde{X}_-)\frac{1}{1-\tilde{T}\tilde{\mathcal{M}}}\tilde{T}\begin{pmatrix}\vec{y}_+ \\ \vec{y}_-\end{pmatrix}\right]$$

where

$$\vec{f} = \{ f_n \}$$

$$\tilde{\mathcal{M}} = \begin{pmatrix} \tilde{X} & \tilde{X}_+ \\ \tilde{X}_- & \tilde{X} \end{pmatrix} \qquad \tilde{\mathcal{T}} = \begin{pmatrix} \tilde{T} & 0 \\ 0 & \tilde{T} \end{pmatrix}$$

It is not hard to prove that

- $f_{2n+1} = 0 \qquad f_{2n} = 1$

This means

$$\Omega = \frac{1}{2}\left(C(i) + C(-i) \right)$$

<u>Midpoint insertion</u> (in twisted theory).

Ghost Neumann coefficients

$$\tilde{V}^{rs}_{nm} = -(-1)^{n+m} \cdot \tilde{N}^{rs}_{nm}$$

where

$$|\hat{0}\rangle = c_0 c_1 |0\rangle$$

$$|\dot{0}\rangle = c_1 |0\rangle$$

$$\tilde{N}^{rs}_{nm} = \langle \tilde{V}_{123} | b^{(r)}_{-n} c^{(s)}_{-m} |\dot{0}\rangle_{123}$$

$$= \oint \frac{dz}{2\pi i} \oint \frac{dw}{2\pi i} \frac{1}{z^{n-1}} \frac{1}{w^{m+2}} \left(f_r'(z)\right)^2 \frac{-1}{f_r(z) - f_s(w)} \cdot$$

$$\cdot \prod_{i=1}^{3} \frac{f_s(w) - \xi_i}{f_r(z) - \xi_i} \left(f_s'(w)\right)^{-1}$$

SL(2,R)-invariant bc propagator

$$\langle b(z)\, c(w) \rangle = \frac{1}{z-w} \prod_{i=1}^{3} \frac{w - \xi_i}{z - \xi_i}$$

We choose

$$\xi_i = f_i(0) = \alpha^{2-i}, \qquad \alpha^3 = 1$$

then

$$\prod_{i=1}^{3} \frac{f_s(w) - f_i(0)}{f_r(z) - f_i(0)} = \frac{f^3(w) - 1}{f^3(z) - 1} \qquad \forall r, s = 1, 2, 3$$

Decomposition

$$\tilde{N}^{rs}_{nm} = \frac{1}{3}\left(\tilde{E}_{nm} + \bar{\alpha}^{r-s}\, \tilde{U}_{nm} + \alpha^{r-s}\, \tilde{\bar{U}}_{nm} \right)$$

$$\tilde{E}_{nm} = \oint \frac{dz}{2\pi i} \oint \frac{dw}{2\pi i} \frac{1}{z^{n+1}} \frac{1}{w^{m+1}} \left(\frac{1}{1+zw} - \frac{w}{w-z} \right)$$

$$\tilde{U}_{nm} = \oint \frac{dz}{2\pi i} \oint \frac{dw}{2\pi i} \frac{1}{z^{n+1}} \frac{1}{w^{m+1}} \left(\frac{1}{1+zw} - \frac{w}{w-z} \right) \frac{f(z)}{f(w)} = (-1)^{n+m} \tilde{\bar{U}}_{nm}$$

Properties

$$\tilde{N}^{rs}_{nm} = \tilde{N}^{r+1,s+1}_{nm}$$

$$\tilde{N}^{rs}_{mn} = (-1)^{n+m}\,\tilde{N}^{sr}_{nm}$$

Ambiguity

$\tilde{N}^{rs}_{nm}$ with $\overset{r=s}{-1 \leq n, m \leq 1}$ are ambiguous

Fix ambiguity:

$$\tilde{N}^{rr}_{-1,1} = \tilde{N}^{rr}_{1,-1} = 0 \qquad N^{rr}_{0,0} = 1$$

Then

$$\boxed{\sum_{k=0}^{\infty} \tilde{U}_{nk}\,\tilde{U}_{km} = \delta_{nm}}$$

Consequences: define

$$\tilde{X}^{rs} = C\,\tilde{V}^{rs}$$

Then

$$\left[\tilde{X}^{rs}, \tilde{X}^{r's'}\right] = 0$$

Call $\mathcal{Y}$ any $\tilde{X}^{rs}$

$$\mathcal{Y} = \left|\begin{array}{c|c} 1 & 0 \\ \hline \vec{y} & Y \end{array}\right| \qquad Y = \{Y_{nm},\ n,m \geq 1\}$$

Set

$$y \equiv \tilde{X}^{11}$$
$$y_+ \equiv \tilde{X}^{12}$$
$$y_- \equiv \tilde{X}^{21}$$

Then

$$y + y_+ + y_- = 1$$
$$y^2 + y_+^2 + y_-^2 = 1$$
$$y_+^3 + y_-^3 = 2y^3 - 2y^2 + 1$$
$$y_- y_+ = y^2 - y$$
$$[y, y_\pm] = 0$$
$$[y_+, y_-] = 0$$

which decompose into

$$Y + Y_+ + Y_- = 1 \qquad \vec{y} + \vec{y}_+ + \vec{y}_- = 0$$
$$Y^2 + Y_+^2 + Y_-^2 = 1 \qquad (1+Y)\vec{y} + Y_+\vec{y}_+ + Y_-\vec{y}_- = 0$$
$$Y_+^3 + Y_-^3 = 2Y^3 - 3Y^2 + 1 \qquad Y_+^2\vec{y}_+ + Y_-^2\vec{y}_- = (2Y^2 - Y - 1)\vec{y}$$
$$Y_+ Y_- = Y^2 - Y \qquad Y_+\vec{y}_- = Y\vec{y} = Y_-\vec{y}_+$$
$$[Y, Y_\pm] = 0 \qquad -Y_\pm\vec{y} = (1-Y)\vec{y}_\pm$$
$$[Y_+, Y_-] = 0$$

Energy of D-25 brane

Total energy = − action (for static config.)

$$S(\Psi) = -\frac{1}{6g_0^2}\langle\Psi_g|Q|\Psi_g\rangle\langle\Psi_m|\Psi_m\rangle$$

For the sliver

$$\langle\Psi_m|\Psi_m\rangle = \langle 0|0\rangle\left(\frac{\mathrm{Det}(1-\Sigma\mathcal{V})}{\sqrt{\det(1-S^2)}}\right)^D$$

$$\langle\Psi_g|Q|\Psi_g\rangle = \frac{\det(1-\tilde{S}^2)}{[\mathrm{Det}(1-\tilde{\Sigma}\tilde{\mathcal{V}})]^2} \qquad \langle 0|0\rangle = \frac{V^{(D)}}{(2\pi)^D}$$

To evaluate these determinants ⟶ to know large L (level) behaviour.

Eigenvalue density $\qquad \rho(k) \approx \frac{1}{2\pi}\log L + \rho_{fin}(k)$

$L \to \infty$

Then

$$\det(1+T) = h_+ L^{-1/3} + \cdots$$

$$\det(1-T) = h_- L^{1/6} + \cdots$$

$$\det(1-X) = h_X L^{1/9} + \cdots$$

$$\det(1+\tilde{T}) = \tilde{h}_+ L^{1/6} + \cdots$$

$$\det(1-\tilde{X}) = \tilde{h}_X L^{-5/9}$$

Then

$$\text{D25-Total Energy} \approx V^{(26)} L^{-\frac{5}{36}D} \cdot L^{\frac{11}{18}} \xrightarrow[L \to \infty]{} 0$$

This could be made up for by defining

$$S[\Psi] = -K\left(\frac{1}{2}\langle \Psi \mathcal{Q} \Psi \rangle + \frac{1}{3}\langle \Psi * \Psi * \Psi \rangle\right) \qquad K \to \infty$$

But, with a field redefinition

$$\hat{\Psi} = e^{\frac{1}{4}\ln\epsilon\,(L_2 - L_{-2})}\Psi \quad , \qquad K = \frac{1}{g_0^2 \epsilon^3}$$

we arrive at

$$S[\Psi] = -\frac{1}{g_0^2}\left(\frac{1}{2}\langle \hat{\Psi} \mathcal{Q} \hat{\Psi} \rangle + \frac{1}{3}\langle \hat{\Psi} * \hat{\Psi} * \hat{\Psi} \rangle\right)$$

But, if

$$\epsilon = g_0^{2/3}$$

then

$$S[\Psi] = -\left(\frac{1}{2}\langle \Psi \mathcal{Q} \Psi \rangle + \frac{1}{3}\langle \Psi | \Psi * \Psi \rangle\right)$$

No freedom left!

This possible only for $D = 26$ in VSFT

$D \neq 26$ anomalies!

Dressed Sliver

Introduce projectors ρ_1, ρ_2 in Fock space

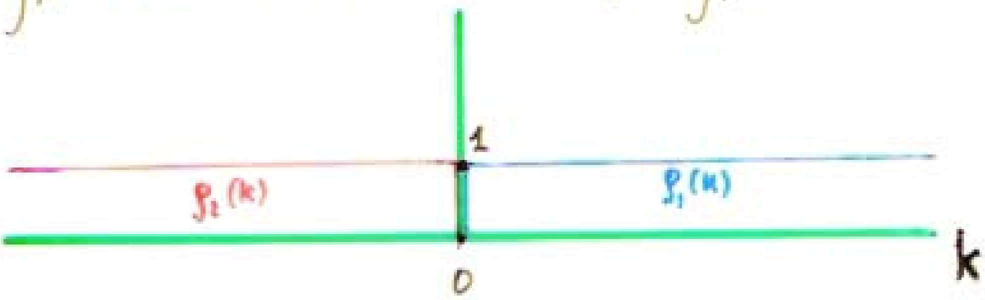
$$\rho_1^2 = \rho_1 \quad , \quad \rho_2^2 = \rho_2 \quad , \quad \rho_1 + \rho_2 = 1$$

$$\rho_1 = \frac{1}{(1+T)(1-X)}\left[X^{12}(1-TX) + T(X^{21})^2\right] \qquad X^{ij} = CV^{ij}$$

$$\rho_2 = \frac{1}{(1+T)(1-X)}\left[X^{21}(1-TX) + T(X^{12})^2\right]$$

In k-basis

$$\rho_1 = \int \rho_1(k)\, |k\rangle\langle k|\, dk \qquad \rho_2 = \int \rho_2(k)\, |k\rangle\langle k|\, dk$$



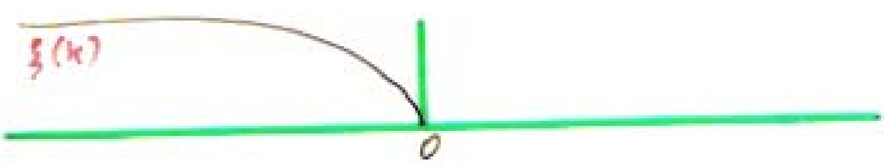
Define vector $\xi = \{\xi_n\}$ s.t. $n = 1, 2, \dots \infty$

$$\rho_2 \xi = \xi \quad , \quad \rho_1 \xi = 0$$

$$\langle \xi | \frac{1}{1-T^2} | \xi \rangle = 1, \qquad \langle \xi | \frac{T}{1-T^2} | \xi \rangle = \kappa$$

In k-basis

$$\langle \xi | = \int dk \; \xi(k) \langle k | \qquad -1 \le \kappa \le 0$$



Now define

$$R = \frac{1}{K+1}\left(|\xi\rangle\langle C\xi| + |C\xi\rangle\langle\xi|\right)$$

$$R_{nm} = \frac{1}{K+1}\left(\xi_n(-1)^m\xi_m + \xi_n(-1)^n\xi_m\right)$$

and a '*deformed*' sliver

$$|\hat{\Xi}\rangle = \hat{\mathcal{N}}\, e^{-\frac{1}{2}a^\dagger \hat{S} a^\dagger}|0\rangle$$

$$\hat{S} = S + R$$

Then

$$T \to \hat{T} = T + P \qquad\qquad P = CR$$

$$P = \frac{1}{K+1}\left(|\xi\rangle\langle\xi| + |C\xi\rangle\langle C\xi|\right)$$

One can prove that

$$|\hat{\Xi}\rangle * |\hat{\Xi}\rangle = |\hat{\Xi}\rangle$$

$|\hat{\Xi}\rangle$ is the **DRESSED SLIVER.**

Now we deform the **DS**

$$\hat{S} \to \hat{S}_\epsilon = S + \epsilon R$$

$$\hat{T} \to \hat{T}_\epsilon = T + \epsilon P$$

So

$$|\hat{\Xi}\rangle \to |\hat{\Xi}_\epsilon\rangle = \hat{\mathcal{N}}_\epsilon\, e^{-\frac{1}{2}a^\dagger \hat{S}_\epsilon a^\dagger}|0\rangle$$

In general

$$|\hat{\Xi}_\epsilon\rangle * |\hat{\Xi}_\epsilon\rangle \neq |\hat{\Xi}_\epsilon\rangle$$

We use ϵ to regularize the action. For instance

$$\det\left(1-\hat{S}_\epsilon^2\right) = \det\left(1-T^2\right)\,(1-\epsilon)^2\left(\frac{2}{K+1}\right)^2$$

Now do the same for the ghost part $|\hat{\tilde{\Xi}}\rangle \to |\hat{\tilde{\Xi}}_\epsilon\rangle$.

Put all together, requiring

$$1-\epsilon = s\, L^{-\frac{5}{36}}$$

$$1-\tilde{\epsilon} = \tilde{s}\, L^{-\frac{11}{36}}$$

Then

$$\text{Tension} = -\frac{S[\hat{\Psi}]}{V^{(D)}} = \frac{1}{6 g_0^2 (2\pi)^D}\,\frac{(\hat{K}+1)^2}{(K+1)^D}\,\frac{\tilde{s}^2}{s^D}\,\frac{h^{D/2}}{\tilde{h}}$$

D-brane spectrum

If Φ_0 is a classical solution, small fluctuations have to satisfy the linearized eq. of motion (LEOM)

$$\mathcal{Q}\phi + \Phi_0 * \phi + \phi * \Phi_0 = 0$$

Now suppose Φ_0 is the dressed sliver $\Phi_0 = \hat{\Xi} \otimes \hat{\tilde{\Xi}}$

Choose

$$|\hat{\phi}_e(t,p)\rangle = \mathcal{N}_e\, e^{-t a^\dagger \hat{p}} |\hat{\Xi}_e\rangle \otimes |\hat{\tilde{\Xi}}\rangle\, e^{ipx} \equiv |\hat{\varphi}_e\rangle \otimes |\hat{\tilde{\Xi}}\rangle$$

It must satisfy

$$\mathcal{Q}|\hat{\tilde{\Xi}}\rangle + |\hat{\tilde{\Xi}}\rangle *_g |\hat{\tilde{\Xi}}\rangle = 0$$

$$|\hat{\varphi}_e\rangle = |\hat{\Xi}\rangle *_m |\hat{\varphi}_e\rangle + |\hat{\varphi}_e\rangle *_m |\hat{\Xi}\rangle$$

These are satisfied provided

$$\boxed{t = t_0 + \alpha \langle \xi | \frac{1}{1+T} | t_0\rangle (1+C)\xi + \beta (1-C)\xi}$$

where

$$|t_0\rangle = 3\,\frac{T^2 - T + 1}{1+T} |v_0\rangle \qquad |v_0\rangle_m = V^{11}_{0n}$$

and α, β arbitrary. Moreover

$$\boxed{e^{-\frac{1}{2}Gp^2} = \frac{1}{2}}$$

Naively: $G = 0$!!

After regularization

$$G = 2 \ln 2$$

Therefore $p^2 = 1 \Rightarrow M^2 = -1 \Rightarrow$ tachyon

<u>Next fluctuation</u>: $\rho_1 \xi = 0\,,\quad \rho_2 \xi = \xi \qquad \xi = \{\xi_n\}$

$$|\hat{\varphi}_v(d^\mu, t, p)\rangle = \mathcal{N}_v \, d^\mu \langle (1-C)\xi \, a^+_\mu \rangle \, |\hat{\varphi}(t,p)\rangle$$

One gets

$$|\hat{\varphi}_v\rangle * |\hat{\Xi}\rangle + |\hat{\Xi}\rangle * |\hat{\varphi}_v\rangle = e^{-\frac{1}{2} G p^2} \Big[d^\mu \langle (1-C)\xi \, a^+_\mu \rangle +$$

$$+ \frac{1}{\kappa+1} \langle \xi | \frac{1}{1-T} | \xi \rangle \, d^\mu \langle (1-C)\xi \, a^+_\mu \rangle + 2\beta \; p\cdot d \, \langle \xi | \frac{\kappa - T}{1-T^2} | \xi \rangle \Big] \mathcal{N}_v \, |\hat{\varphi}(t,p)\rangle$$

It must be

$p^2 = 0$ massless

$p \cdot d = 0$ transversality

$\langle \xi | \frac{1}{1-T} | \xi \rangle = 0$ aux. cond. on ξ

This represent the <u>vector fluctuation</u>.

At level 2:

$$|\hat{\varphi}(\theta_1, \theta_2, t, p)\rangle = \theta_1^{\mu_1} \langle a^+_{\mu_1} \xi_1^{(1)} \rangle |\hat{\varphi}(t,p)\rangle + \theta_2^{\mu_1 \mu_2} \langle a^+_{\mu_1} \xi_1^{(2)} \rangle \langle a^+_{\mu_2} \xi_2^{(2)} \rangle |\hat{\varphi}(t,p)\rangle$$

In order for this to be a solution

$$p^2 = -1, \qquad \kappa^2 = 1$$

$+$ Virasoro constraints

etc.