Branes and brane decay in SFT

Zlatibor 2004

- A challenge from Cosmology
- String theory and SFT
- Branes in SFT (VSFT)
- Brane decay: time-localized solutions in VSFT

Table 3. "Best" Cosmological Parameters

Description	Symbol	Value	+ uncertainty	− uncertainty
Total density	Ω_{tot}	1.02	0.02	0.02
Equation of state of quintessence	w	< -0.78	95% CL	—
Dark energy density	Ω_Λ	0.73	0.04	0.04
Baryon density	$\Omega_b h^2$	0.0224	0.0009	0.0009
Baryon density	Ω_b	0.044	0.004	0.004
Baryon density (cm^{-3})	n_b	2.5×10^{-7}	0.1×10^{-7}	0.1×10^{-7}
Matter density	$\Omega_m h^2$	0.135	0.008	0.009
Matter density	Ω_m	0.27	0.04	0.04
Light neutrino density	$\Omega_\nu h^2$	< 0.0076	95% CL	—
CMB temperature (K)[a]	T_{cmb}	2.725	0.002	0.002
CMB photon density (cm^{-3})[b]	n_γ	410.4	0.9	0.9
Baryon-to-photon ratio	η	6.1×10^{-10}	0.3×10^{-10}	0.2×10^{-10}
Baryon-to-matter ratio	$\Omega_b \Omega_m^{-1}$	0.17	0.01	0.01
Fluctuation amplitude in $8h^{-1}$ Mpc spheres	σ_8	0.84	0.04	0.04
Low-z cluster abundance scaling	$\sigma_8 \Omega_m^{0.5}$	0.44	0.04	0.05
Power spectrum normalization (at $k_0 = 0.05\ \mathrm{Mpc}^{-1}$)[c]	A	0.833	0.086	0.083
Scalar spectral index (at $k_0 = 0.05\ \mathrm{Mpc}^{-1}$)[c]	n_s	0.93	0.03	0.03
Running index slope (at $k_0 = 0.05\ \mathrm{Mpc}^{-1}$)[c]	$dn_s/d\ln k$	−0.031	0.016	0.018
Tensor-to-scalar ratio (at $k_0 = 0.002\ \mathrm{Mpc}^{-1}$)	r	< 0.90	95% CL	—
Redshift of decoupling	z_{dec}	1089	1	1
Thickness of decoupling (FWHM)	Δz_{dec}	195	2	2
Hubble constant	h	0.71	0.04	0.03
Age of universe (Gyr)	t_0	13.7	0.2	0.2
Age at decoupling (kyr)	t_{dec}	379	8	7
Age at reionization (Myr, 95% CL))	t_r	180	220	80
Decoupling time interval (kyr)	Δt_{dec}	118	3	2
Redshift of matter-energy equality	z_{eq}	3233	194	210
Reionization optical depth	τ	0.17	0.04	0.04
Redshift of reionization (95% CL)	z_r	20	10	9
Sound horizon at decoupling (°)	θ_A	0.598	0.002	0.002
Angular size distance (Gpc)	d_A	14.0	0.2	0.3
Acoustic scale[d]	ℓ_A	301	1	1
Sound horizon at decoupling (Mpc)[d]	r_s	147	2	2

[a] from *COBE* (Mather et al. 1999)

[b] derived from *COBE* (Mather et al. 1999)

[c] $l_{eff} \approx 700$

[d] $\ell_A \equiv \pi \theta_A^{-1}$ $\quad \theta_A \equiv r_s\, d_a^{-1}$

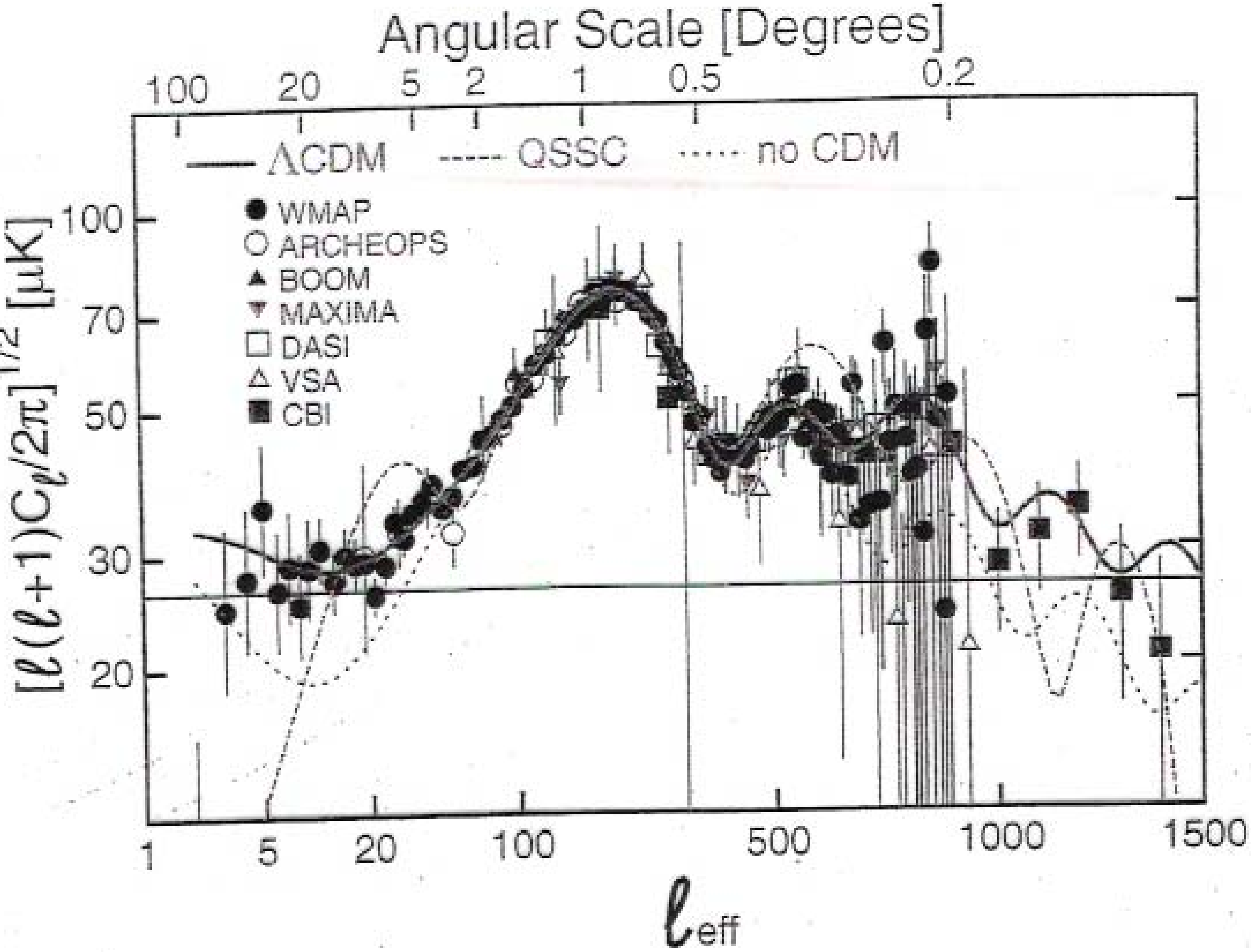
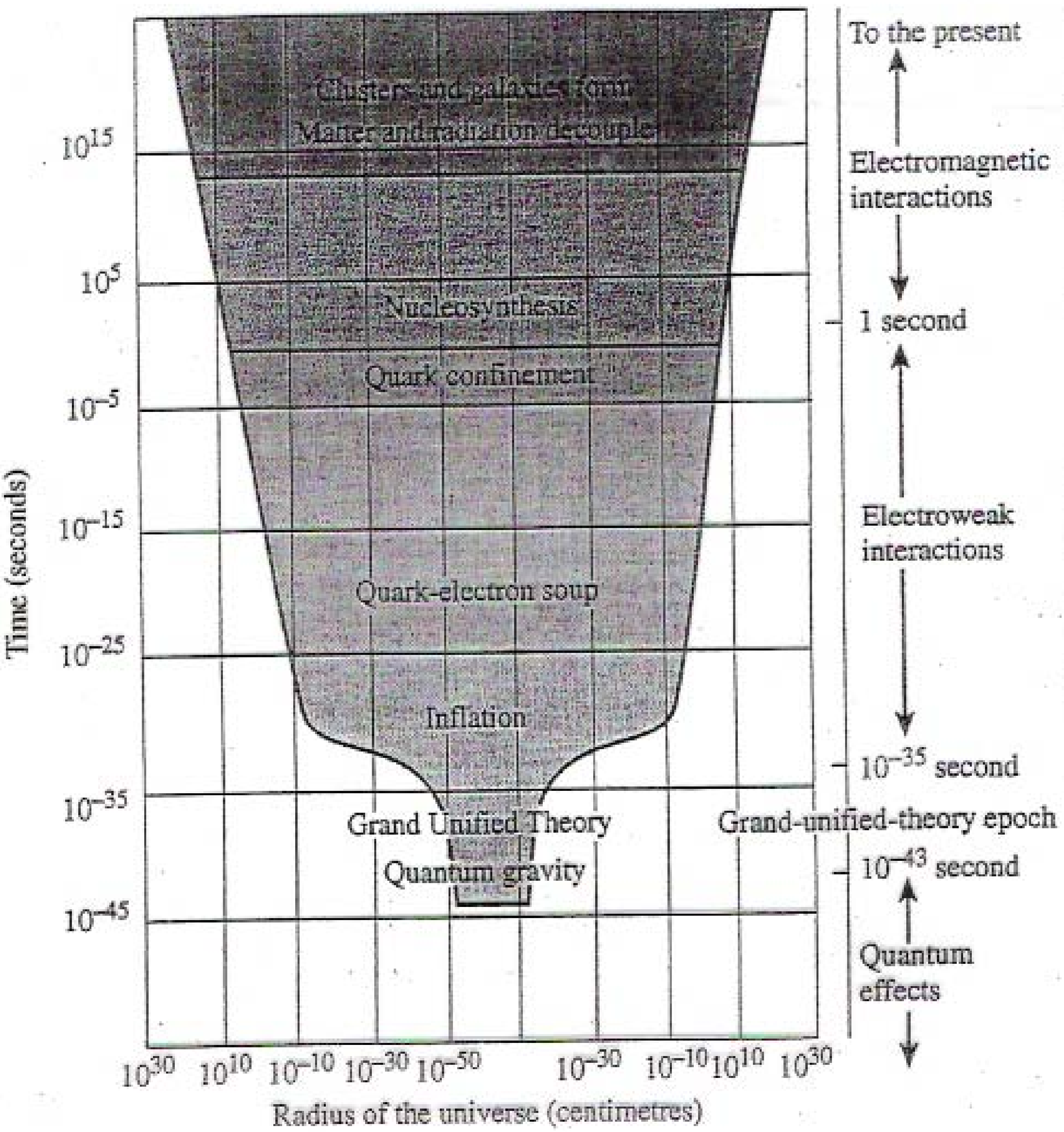


Fig. 3.— The angular power spectrum of the CMB from *WMAP* and earlier balloon-borne and ground-based data. The ΛCDM model is a good fit to all of these datasets. The Quasi-Steady-State cosmology and the no-CDM model inspired by MOND (Modification Of Newtonian Dynamics) both give unacceptable best fits to the CMB angular power spectrum, with large deviations in the low ℓ region observed by *COBE* and confirmed by *WMAP*.

Time	Particle Physics	Cosmological Event
10^{-43} s	String Theory?	Gravitons decouple ?
10^{-43} s - 10^{-12} s	Grand Unification? Desert? String Theory? Extra dimensions?	Topological defects? Baryogenesis? Inflation?
10^{-12} s	Electroweak Breaking	Baryogenesis?
10^{-5} s	QCD scale	Quark-Hadron transition
$10^{-2} - 10^{2}$ s	Nuclear Physics scale	Nucleosynthesis, Neutrinos decouple
10^{11} s	Atomic Physics scale	Atoms formed, CMB Matter domination

The very early universe



FRW cosmology

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right]$$

$k = +1, 0, -1$

Insert into Einstein eq.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G\, T_{\mu\nu} - \Lambda g_{\mu\nu}$$

with

$$T_{00} = \rho\ , \qquad T_{ij} = p\, g_{ij}$$

We get

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

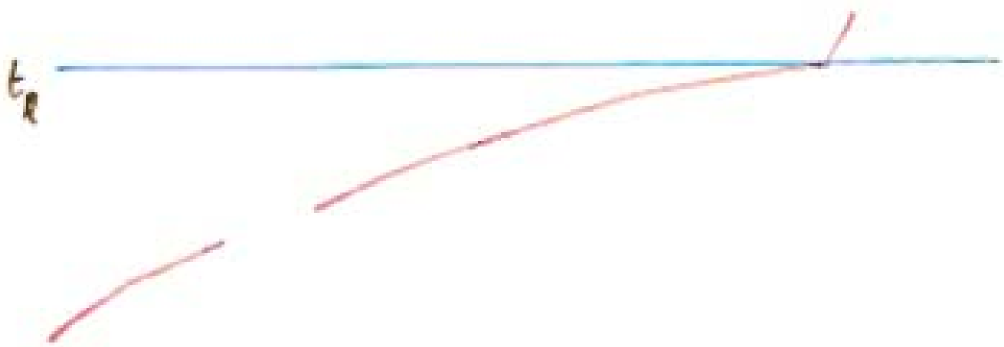
$H = \frac{\dot{a}}{a}$

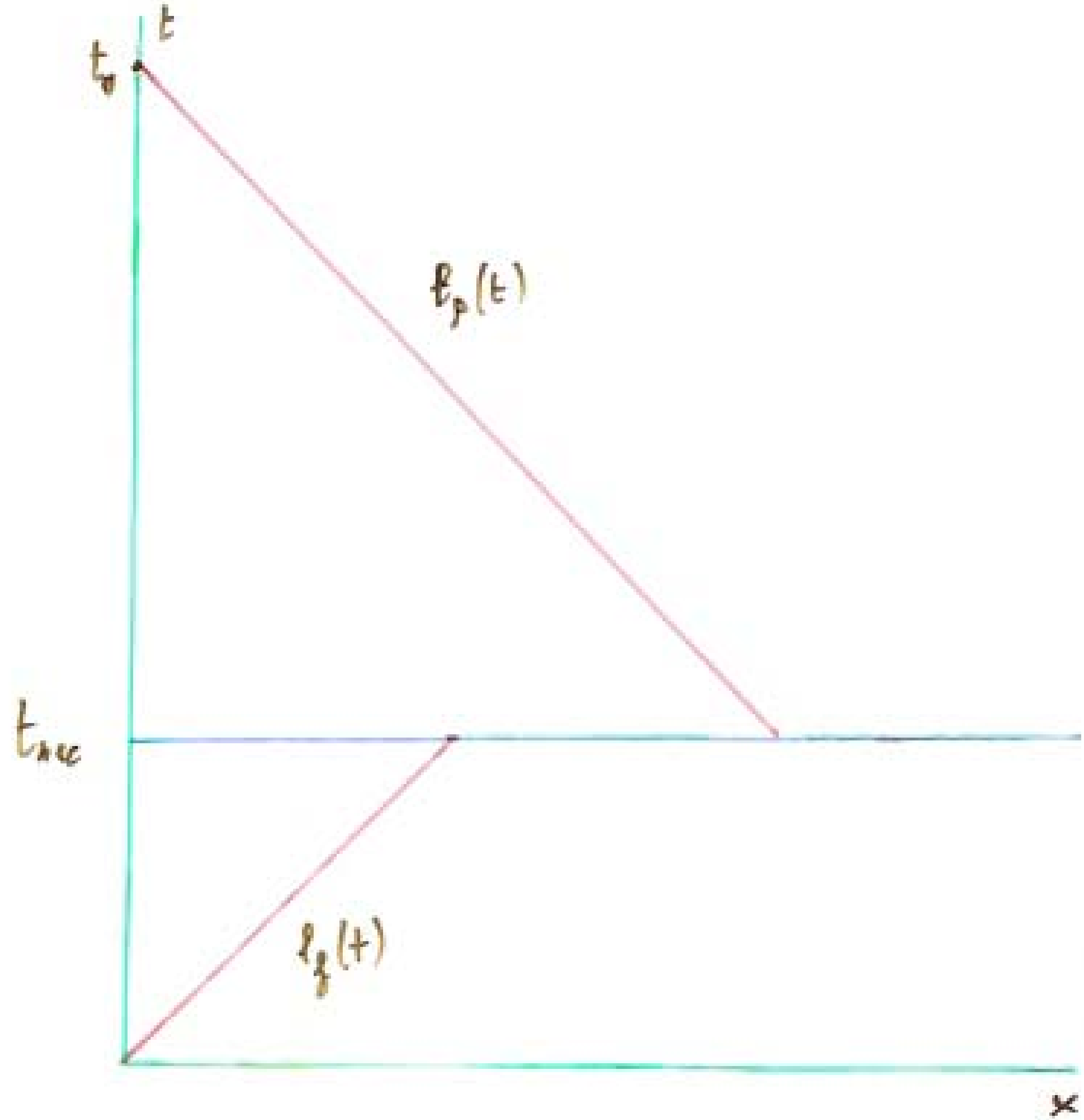
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p\right) + \frac{\Lambda}{3}$$

which entail energy conservation

$$\dot{\rho} = -3H\left(\rho + p\right)$$

Matter	$\rho \sim a^{-3}$	$a(t) \sim t^{2/3}$	
Radiation	$\rho \sim a^{-4}$	$a(t) \sim t^{1/2}$	$k=0$
Vacuum	$\rho \sim \frac{\Lambda}{8\pi G}$	$a(t) \sim e^{\sqrt{\Lambda/3}\,t}$	




t
t_0
$\ell_p(t)$
t_{acc}
$\ell_f(t)$
x

Inflation $\Longleftrightarrow$ $\ddot{a} > 0$

for $\Lambda = 0$ $\Longrightarrow$ $\rho + 3p < 0$

Model for inflation: scalar field ϕ (inflaton)
with potential $V(\phi)$

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$
$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

Then

$$H^2 = \frac{8\pi G}{3}\left[V(\phi) + \frac{1}{2}\dot{\phi}^2\right] \qquad k = \Lambda = 0$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

"Slow-roll" conditions

$$\epsilon = \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1$$
$$\eta = M_P^2 \frac{V''}{V} \ll 1.$$

Number of e-foldings

$$N_e = \int_{t_{init}}^{t_{fin}} H(t')\,dt' = \frac{1}{M_P^2}\int_{\phi_{fin}}^{\phi_{in}} \frac{V}{V'}\,d\phi$$

Inflation explains:

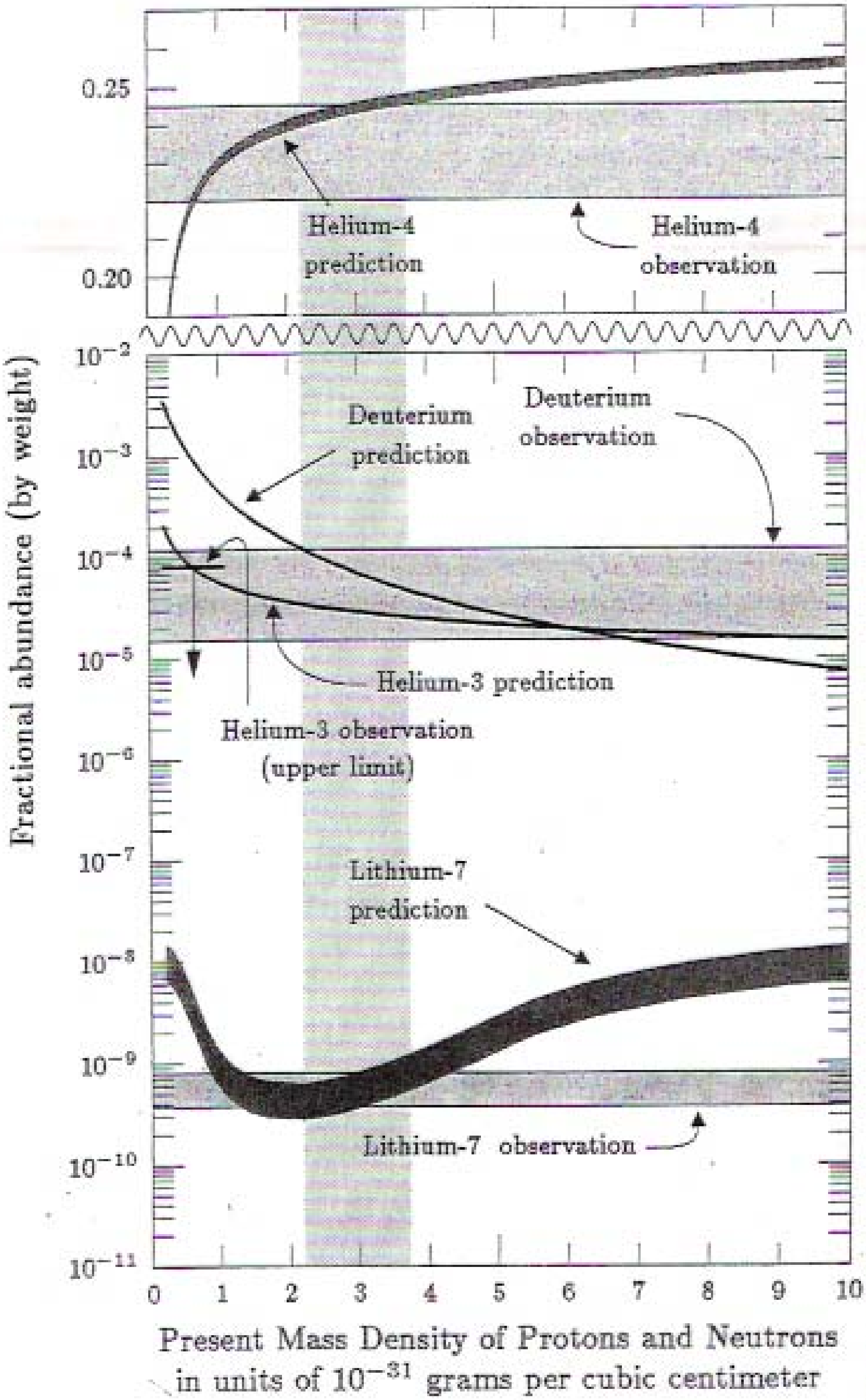
- homogeneity of universe in the large
- flatness
- no monopoles

How does inflation stops? graceful exit

$$V(x,y) = a\left(y^2-1\right)x^2 + b\,x^4 + c$$

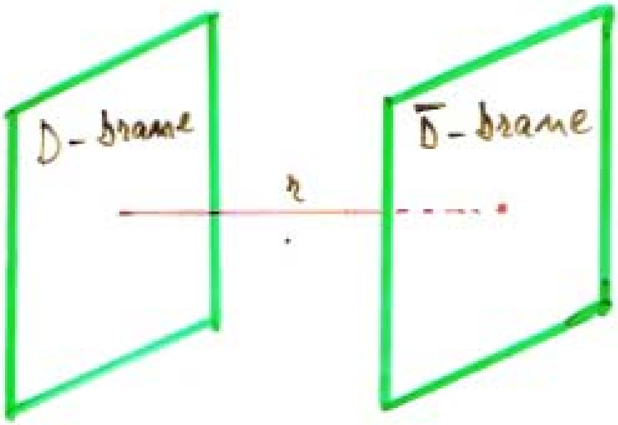
From inflation $\longrightarrow$ inhomogeneity of CMB

$\longrightarrow$ abundance of He and H_2



0.25
0.20
Helium-4 prediction
Helium-4 observation
Fractional abundance (by weight)
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
Deuterium prediction
Deuterium observation
Helium-3 prediction
Helium-3 observation (upper limit)
Lithium-7 prediction
Lithium-7 observation
0 1 2 3 4 5 6 7 8 9 10
Present Mass Density of Protons and Neutrons in units of 10^{-31} grams per cubic centimeter

Inflation and String theory



r inflaton

transverse volume $\sim L^6$

Potential: $$V(r) = 2T_3\left(1 - \frac{1}{2\pi^3}\frac{T_3}{M_p^8\, r^4}\right)$$

Unfortunately $\eta \ll 1$ requires $r > L$.

Need warping factor (AdS_5 background)

$$ds^2 = \frac{r^2}{R^2}\left(-dt^2 + d\vec{x}^{\,2}\right) + \frac{R^2}{r^2}dr^2$$

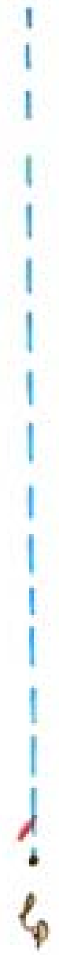
In this background ϵ and η can be taken

$$\epsilon \ll 1 \quad , \quad \eta \ll 1 \quad ,$$

Moreover

$$N_e \sim 60 \quad , \quad \delta_H = \frac{1}{\sqrt{75}\,\pi}\frac{1}{M_p^3}\frac{V^{3/2}}{V'} \approx 1.9 \cdot 10^{-5}$$

!! However stabilization creates mass for inflaton and destroys $\eta \ll 1$

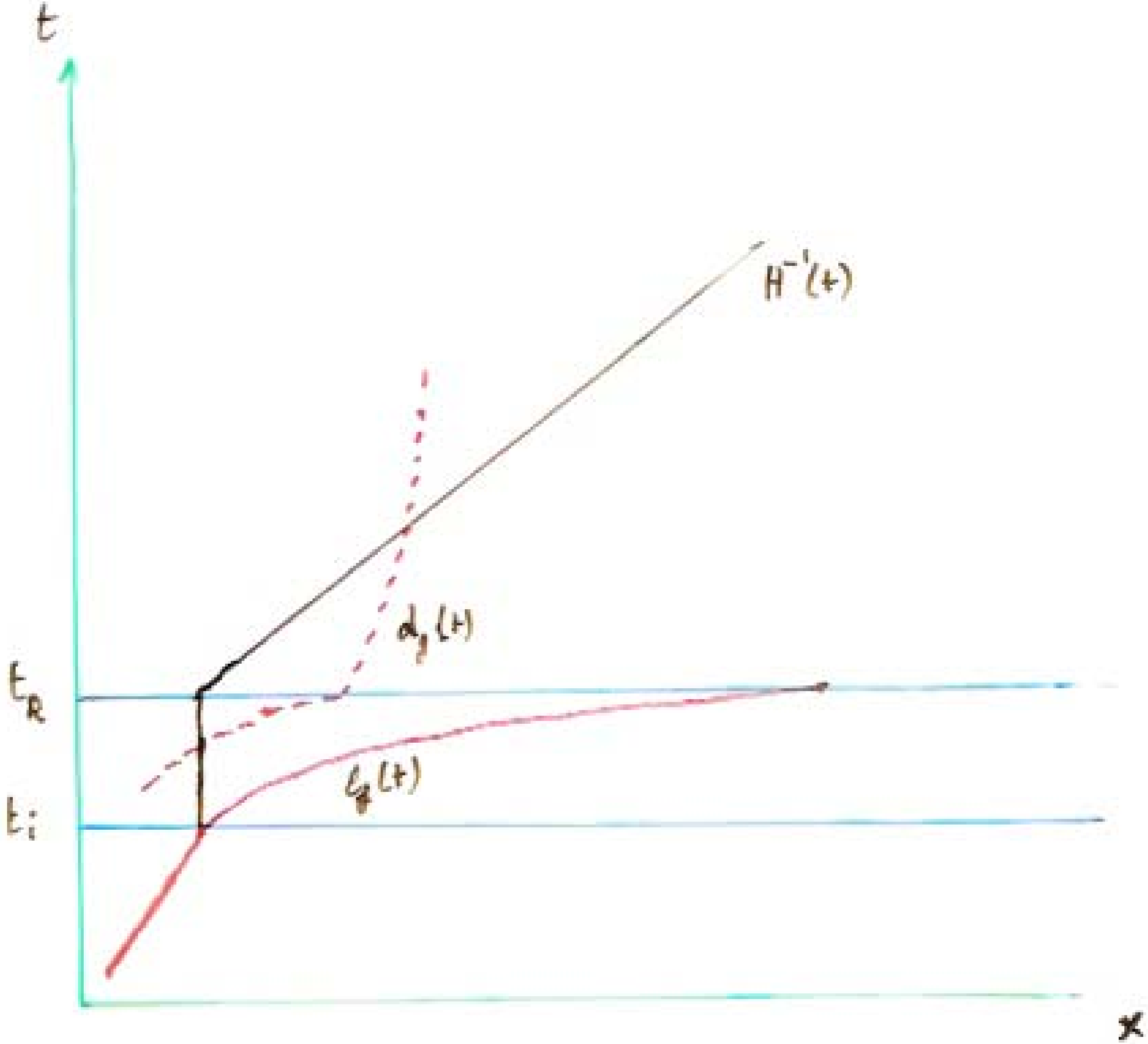


$$M_p = \frac{1}{\sqrt{8\pi G}} = 4.342 \times 10^{-6}\,g = 2.436 \times 10^{18}\ \mathrm{GeV}$$

$$m_p = 1.22 \times 10^{19}\ \mathrm{GeV} = \frac{1}{\sqrt{\alpha'}}$$

$$L_{pl} \equiv \frac{\hbar}{c} M_p = 8.10 \times 10^{-33}\ \mathrm{cm}$$

$$T_{pl} \equiv \frac{\hbar}{c^2} M_p = 2.70 \times 10^{-43}\ \mathrm{sec}$$


t
$H^{-1}(t)$
$d_g(t)$
t_R
$\ell_g(t)$
t_i
x

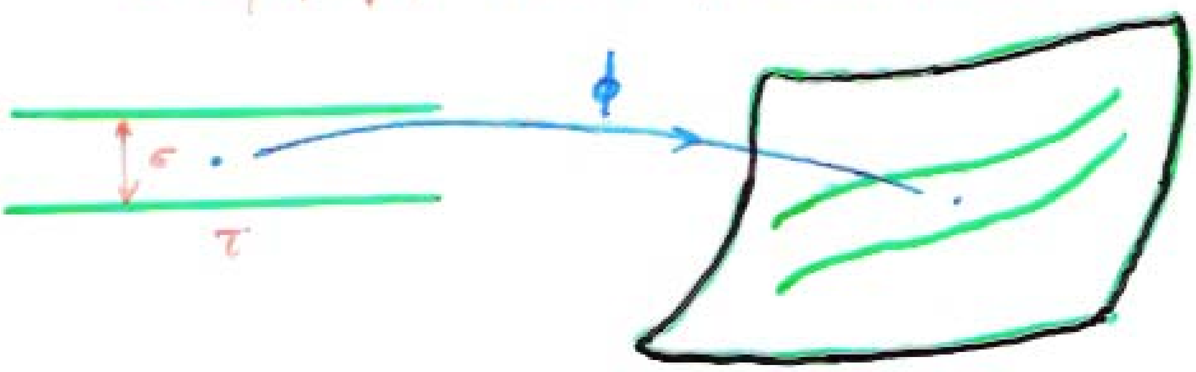
Open String Theory $(D=26)$

Action:

$$S = -\frac{T}{2}\int_{\Sigma} d^2\sigma \, \sqrt{h}\, h^{\alpha\beta}(\sigma)\, \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}$$

where

$$\Sigma = \{\sigma, \tau\}, \quad 0 \le \sigma \le \pi, \quad -\infty < \tau < +\infty$$



$$X^\mu(\sigma,\tau) = X^\mu(\phi(\sigma,\tau))$$

$$T = \frac{1}{2\pi\alpha'} \qquad\qquad l_p \sim \sqrt{\alpha'} \sim 1.6 \times 10^{-33}\,\text{cm}$$

Symmetries:

Reparametrization

$$\xi^\alpha \equiv \xi^\alpha(\sigma,\tau)$$

$$\delta X^\mu = \xi^\alpha \partial_\alpha X^\mu$$

$$\delta h^{\alpha\beta} = \xi^\gamma \partial_\gamma h^{\alpha\beta} - \partial_\gamma \xi^\alpha h^{\gamma\beta} - \partial_\gamma \xi^\beta h^{\alpha\gamma}$$

Weyl

$$\delta X^\mu = 0$$

$$\delta h_{\alpha\beta} = \Lambda(\sigma,\tau)\, h_{\alpha\beta}$$

Equation of motion

$$\Box X^\mu = 0$$

Constraints

$$T_{\alpha\beta} = -\frac{2}{T}\frac{1}{\sqrt{h}}\frac{\delta S}{\delta h^{\alpha\beta}} = \left(\partial_\alpha X^\mu \partial_\beta X^\nu - \frac{1}{2} h_{\alpha\beta} h^{\alpha'\beta'} \partial_{\alpha'} X^\mu \partial_{\beta'} X^\nu\right)\eta_{\mu\nu} = 0$$

Boundary conditions

$$\partial_\sigma X^\mu \, \delta X^\nu \eta_{\mu\nu} \Big|_{\sigma = 0,\pi} = 0$$

N (Neumann) $\quad \partial_\sigma X^\mu = 0$

at $\sigma = 0, \pi$

D (Dirichlet) $\quad X^\mu = \text{const}$

Fix conformal gauge

$$h_{\alpha\beta} = e^\phi \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

EOM

$$\left(\frac{\partial^2}{\partial\sigma^2} - \frac{\partial^2}{\partial\tau^2}\right) X^\mu(\sigma,\tau) = 0$$

General solution

$$X^\mu(\sigma,\tau) = x^\mu + 2\alpha' p^\mu \tau + i\sqrt{2\alpha'} \sum_{n\neq 0}^{\infty} \frac{\alpha_n^\mu}{n} e^{-in\tau} \cos n\sigma$$

$$\alpha_0^\mu = \sqrt{2\alpha'}\, p^\mu$$

Classical constraints $\left(T(z) = \sum_n L_n z^{-n-2}\right)$

$$L_n = 0 \qquad\qquad L_m = \frac{1}{2} \sum_k \alpha^{\mu}_{m-k} \, \alpha^{\nu}_{k} \, \eta_{\mu\nu}$$

become quantum constraints:

$$L_n |\phi\rangle = 0 \qquad n > 0$$

$$L_0 |\phi\rangle = |\phi\rangle$$

where

$$L_n = \frac{1}{2} \sum_k :\alpha_{n-k} \cdot \alpha_k:$$

Alternatively (BRST formulation) define

$$Q_B = \sum_{n=-\infty}^{+\infty} c_n \left(L_{-n} + \frac{1}{2} L^{(gh)}_{-n} \right) - c_0$$

so that

$$Q_B = \sum_n c_n L_{-n} + \sum_{n,k} \frac{m-k}{2} :c_m c_k b_{-m-k}: - c_0$$

and impose

$$Q_B |\psi\rangle = 0 \qquad\qquad |\psi\rangle \neq Q_B |\chi\rangle$$

since

$$Q_B^2 = 0 \quad \text{in} \quad D = 26$$

$Q_B^2 = 0$ comes from

$$[L_n, L_m] = (n-m) L_{n+m} + \frac{c}{12}(n^3 - n)\, \delta_{n+m,0}$$

and

$$c = D \quad \text{for} \quad X^\mu$$

$$c = -26 \quad \text{for} \quad b, c$$

Spectrum: $\alpha' M^2 = -1 + \sum_{n=1}^{\infty} \alpha^\mu_{-n} \alpha^\nu_n \eta_{\mu\nu}$

tachyon	$\vert 0\rangle e^{ikx}$	$M^2 = -\frac{1}{\alpha'}$	
vector	$\xi_\mu \alpha^\mu_{-1} \vert 0\rangle e^{ikx}$	$M^2 = 0$	$\xi \cdot k = 0$
tensor	$\theta_{\mu\nu} \alpha^\mu_{-1} \alpha^\nu_{-1} \vert 0\rangle e^{ikx}$	$M^2 = \frac{1}{\alpha'}$	
$\vdots$	$\vdots$	$\vdots$	

Vertex Operators

$$V_T = \int_{\partial\Sigma} dz\; e^{ik\cdot X}$$

$$V_A = \int_{\partial\Sigma} dz\; A_\mu \partial X^\mu e^{ikX}$$

$\vdots$

Amplitudes (on-shell)

$$\langle V_1 \cdots V_N \rangle \qquad k_i^2 = -M_i^2$$

Examples: 4-tachyon amplitude

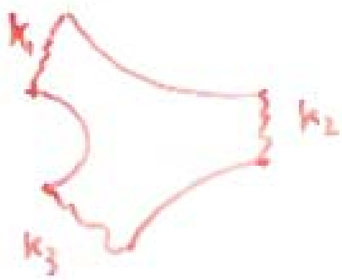
$$A_4(k_1, \dots, k_4) = g^2 \int_0^1 dz\, (1-z)^{k_2 \cdot k_3}\, z^{k_3 \cdot k_4}$$

(Veneziano amplitude)

3-vectors vertex

$$\langle V^{a_1}_{\xi_1}(k_1)\, V^{a_2}_{\xi_2}(k_2)\, V^{a_3}_{\xi_3}(k_3) \rangle = g\, \xi_1^{\mu_1} \xi_2^{\mu_2} \xi_3^{\mu_3}\, t_{\mu_1 \mu_2 \mu_3}\, f^{a_1 a_2 a_3}$$

$$t_{\mu_1\mu_2\mu_3} = (k_2-k_3)_{\mu_1}\eta_{\mu_2\mu_3} + (k_3-k_1)_{\mu_2}\eta_{\mu_1\mu_3} + (k_1-k_2)_{\mu_3}\eta_{\mu_1\mu_2}$$
$$+ \frac{\alpha'}{2}(k_2-k_3)_{\mu_1}(k_3-k_1)_{\mu_2}(k_1-k_2)_{\mu_3}$$



LEEA:

$$S = \frac{1}{g^2}\int d^{26}x \left[-\frac{1}{4}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) - \frac{2}{3} i\alpha' \,\mathrm{Tr}\left(F_\mu{}^\nu F_\nu{}^\lambda F_\lambda{}^\mu\right)\right]$$

LEEA's can be reconstructed to some extent.

When off-shell information is needed $\Rightarrow$ SFT

String theory has two expansion parameters:

α' $\qquad \langle V_1 \cdots V_N \rangle = \langle V_1 \cdots V_N \rangle_0 + \alpha' \langle V_1 \cdots V_N \rangle_1 + \cdots$

g_s $\qquad \langle V_1 \cdots V_N \rangle = \langle V_1 \cdots V_N \rangle_{disk} + g_s \langle V_1 \cdots V_N \rangle_{annulus} + \cdots$

Closed String Theory

Two sets of oscillators : α_m^μ , $\tilde{\alpha}_m^\mu$

Boundary conditions : periodicity

Two copies of Virasoro generators : L_m , $\tilde{L}_m$

Spectrum :

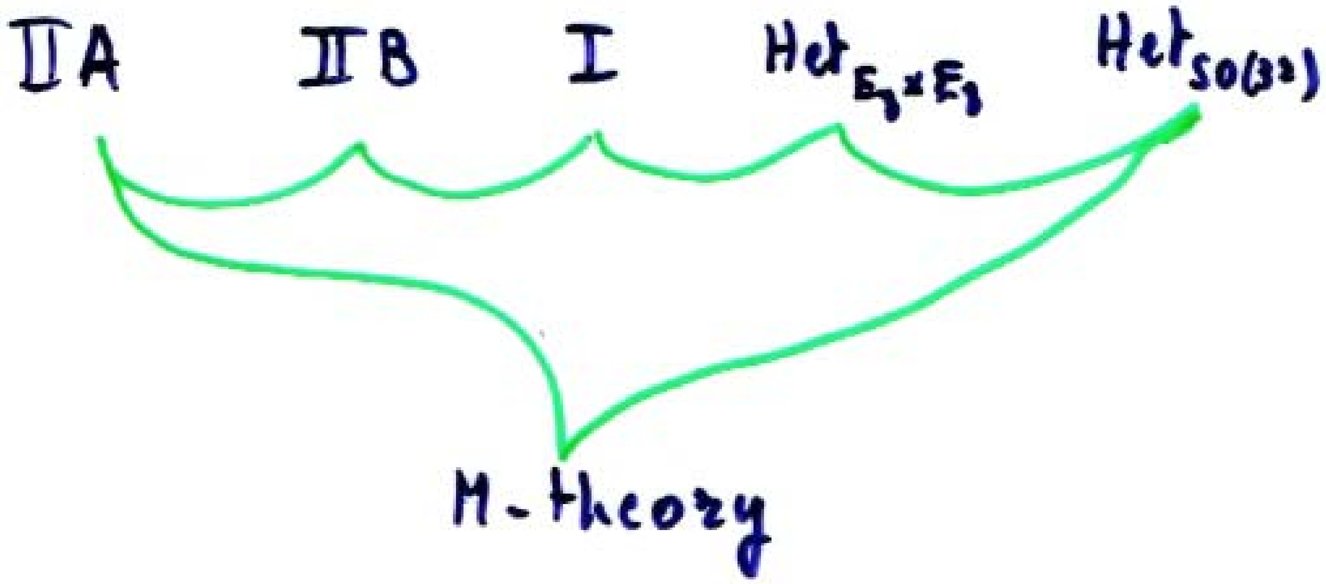
tachyon $\quad |0\rangle e^{ikx} \quad M^2 = -\frac{4}{\alpha'}$

gravity modes $\quad \alpha_{-1}^\mu \tilde{\alpha}_{-1}^\nu |0\rangle e^{ikx} \quad M^2 = 0$

$$\downarrow$$

$$\phi \, , \, G^{\mu\nu} \, , \, B^{\mu\nu}$$

Superstring Theories



Different repr. of D-branes

- Abstract geometrical: hyperplanes with open strings ending on them
- Classical solutions of LEEA of CST's.
- Boundary states in CFT's

$$|B\rangle \sim e^{-\sum_{n=1}^{\infty} a_n^+ \tilde{a}_n^+} |0\rangle$$

- Squeezed states in OSFT